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PRODUCTIVITY RELATIONSHIPS IN ALBERTA GOVERNMENT TELEPHONES

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled "Productivity Relationships in Alberta Government Telephones" submitted by Robert Gordon Bertram in partial fulfilment of the requirements for the degree of Master of Business Administration.

ABSTRACT

The objective of this thesis is to examine the productivity relationships at Alberta Government Telephones and to further develop methods of measuring these relationships. In the communications industry, fundamental changes in the capital/labour ratio have made it necessary to refine techniques for planning capital and financial expenditures. Applied to these areas, productivity measurements can prove to be supplementary tools for analyzing methods of reducing overall unit costs and predicting future performance.

The above objective is attained by a two stage procedure. First, indexes of partial and total factor productivity are measured and, comparing them with similar indexes developed using Bell Canada data, used to determine whether the company is representative of the industry. Second, the Alberta Government Telephones data is fitted, by indirect linear and by direct non-linear estimation techniques, to the 'constant elasticity of substitution' production function. Parameter estimates of this function, obtained through the use of these techniques, are used to analyze productivity relationships at Alberta Government Telephones.

The results achieved by this model were satisfactory. The parameter estimates of the 'CES' function showed that growth at Alberta Government Telephones was primarily a result of returns to scale and increasing capital intensity.

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CHAPTER I

INTRODUCTION

General

Productivity is a concept that has been given many meanings. Its meaning has been as limited as a measure of the personal efficiency of labour and as broad as measuring the efficiency in deriving output from all resources available. The concept of productivity can be defined as a measurement of the amount of output which can be produced with given factors of production. Given this definition the only operational concept of productivity that can be measured with existing statistical techniques is the one which measures the amount of output that can be achieved with a given amount of inputs. This is a measurement of the average physical product of a factor of production. Such productivity measurements are usually defined in terms of index numbers and are used as a measurement of variations in efficiency. This measurement of the average physical product of a factor of production however is the end result of changes in all of the dynamic variables that contribute to changes in the total physical product. In measuring this end result, productivity indexes do not explicitly show how the dynamic variables have interacted. To understand these relationships another method must be employed.

A more complete understanding of the dynamics involved in the production process can be achieved by relating output to inputs through

a mathematical model such as the 'constant elasticity of substitution' production function. By estimating the production function for an economic unit the contribution of the inputs to the production process as well as the relationships amongst the inputs is more explicitly considered.

Analysis by means of productivity indexes, whether they relate to labour, capital or total productivity, provides only a measurement of the end result of the production process. A production function allows variables such as technological change and their role within the production process to be explicitly considered. Thus the value of productivity indexes is in crystallizing, at a moment in time, the level of growth that has been achieved and the value of a production function is in understanding how the development was achieved.

The starting point for this study is a productivity study done at 'Bell Canada'.[1] The intention is to duplicate a portion of this study using data derived from Alberta Government Telephones. Variations of this data are then used to fit the 'constant elasticity of substitution' production function.

The analysis in this thesis will be presented in the following manner. The next chapter will be devoted to a review of productivity relationships, production functions, concepts and definitions. Particular emphasis will be placed on a review of the relevant theory. Chapter III will be a review of the model to be used in this study. Chapter IV will

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[1] R. E. Olley, Productivity Gains in a Public Utility. (A paper presented to the Annual Meeting of the Canadian Economics Association, Winnipeg, Manitobe, June, 1970).

analyze the results from the productivity indexes and Chapter V will analyze the results of fitting the production function. Chapter VI will present overall conclusions.

Problem Description and Purpose

Alberta Government Telephones has been rapidly transformed from a labour intensive to a capital intensive firm. Some of the repercussions of this transformation can best be seen with the assistance of productivity indexes and by fitting an aggregate production function.

The specific objectives of this study are to first measure productivity indexes. The results of this measurement can then be compared to a previous study prepared by Olley and can act as a measure of past efficiency in the use of factors of production. Second, an attempt will be made to fit the constant elasticity of substitution production function to the available data through the use of multiple regression. Estimates of this function explicitly encompass changes in the parameters for capital intensity, returns to scale, efficiency of production and indirectly the elasticity of substitution.

This study is undertaken to further develop the methods of measuring productivity relationships at Alberta Government Telephones. Intuitively, technological advances taking place in the industry seem to be predominately labour saving. This process combined with the need for rapid expansion to meet growing service demands has made the firm more capital intensive. This fundamental change to a

capital intensive industry has led to a greater need for planning of capital expenditures, financing, budgeting and the measurement of performance. While it will never replace criteria such as least cost, productivity measurements are useful tools of analysis and prediction in all of the decision making areas. A greater understanding of the variables in the production relationship will help in analyzing methods to reduce overall unit costs and in the prediction of future performance. In the future, results achieved in measuring these variables will be employed in simulation models for forecasting future growth and revenues.

Limitations

This study is limited by the data and the limitations of the model employed. Whether the data is used for productivity indexes or for estimating production functions it is explicitly assumed that the output and the inputs are in homogenous categories. However, labour and capital consist of heterogenous elements each with divergent characteristics. This heterogeneity is often considered to be both the cause and result of technical progress and consequently aggregation can affect the magnitude, the stability and the dynamics of total factor productivity.

The models employed in this study are encumbered by some specific weaknesses which will be dealt with in detail at a later point. Because of these weaknesses the proposed models impose certain limitations on the study. In particular the production function explicitly and the productivity indexes implicitly assume that the

parameters of the production function are constant over time. It is also clear that any mis-specification of the production function, for example by omitting some of the relevant factors of production, will spill over into the measure of total factor productivity. Finally, a priori specification of the form and the method of estimation of the production function without any testing of alternatives may lead to a non-optimal approach.

CHAPTER II

CONCEPTS AND THEORIES

Introduction

The purpose of this chapter is to present the theoretical background for the proposed study and to define some of the concepts that will be employed. The first section will discuss productivity, production sets and technological change at an abstract level. The second section will contain theories and concepts pertaining to production functions and the third and fourth sections will discuss the Cobb-Douglas and 'Constant Elasticity of Substitution' functions respectively. The fifth section will present some of the limitations of production functions and the final section will be a summary of the chapter.

The General Production Set

Productivity is both the cause and the result of the many dynamic forces of economic life which are operative in an economy. The theory of production provides methods for the analysis and the understanding of productivity. First, productivity indexes representing the physical product can be developed to measure the past movement of productivity either for the individual factors of production or for all factors of production. This has been pointed out by Salter :

The only significance that can be given to such

> figures is that they are indications of what may be termed 'growth in depth' as distinct from 'extensive growth' - growth which merely reproduces a given situation.[1]

The other method, in which production functions, are used is a general equilibrium relationship for the production process. It is a much more comprehensive attempt at measuring production relationships. Indeed, measurement of productivity indexes requires that they be based in production theory. As Nadiri puts it:

> ...produced either from an explicitly defined production function or from a distribution theory where the production function is implicit.[2]

Production functions are used as a means of representing technological situations either at the firm, at the industry or at some other aggregate level. As Walters States:

> The representation of technological possibilities as a set is the most comprehensive approach to the analysis of production. But, for many practical and for some theoretical applications, this approach is too general to be useful. The set must be restricted and specialized. One such specialized concept is the production function.[3]

Walters further explains that the most general model for explaining technological conditions is activity analysis. He gives the following brief description of such a model:

> The production set is drastically simplified. We first specify that there are a finite number of basic activities. Each basic activity is represented by a number for each

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[1]W. E. G. Salter, Productivity and Technical Change, 2nd ed., (hereinafter referred to as Technical Change), (Cambridge: Cambridge University Press, 1969), p. 3.

[2]M. I. Nadiri, "The Theory and Measurement of Total Factor Productivity", (hereinafter referred to as Theory and Measurement), The Journal of Economic Literature, VIII (Dec. 1970), p. 1140.

[3]A. A. Walters, "Production and Cost Functions". Econometrica, XXXI (Jan. - Apr. 1963), p. 2.

> commodity - a negative value indicating an input and a positive value an output. The technical knowledge of society is subsumed in these basic activities, and in application they are supposed to be constant and reproducible. The activities are assumed to be additive. ...Each activity is independent of others:... The activities are assumed to be infinitely divisible and proportionately reproducible.[4]

Such a production set, described by activity analysis, can be closed by assuming that only limited resources are available. The above model can then be used to describe a bundle of efficient activities.

Turning to technological change, Salter has discussed the relevance of technology and technical knowledge to the production set. He shows that the state or the level of technology places a constraint on the possible production sets. These constraints are derived from within any one of a series of levels ranging from the state of pure science to the extent to which such science is applied in production processes. Salter points out that technique decisions are related to the decisions to purchase new capital so that capital measurements in production functions must be in terms of real investments. From these two assumptions he concludes the following:

> Production functions defined in these terms describe the alternative techniques at each date, and a series of production functions describes the new alternatives opened up by a flow of new technical knowledge. Such a series of production functions may be related to productivity and provide a tool for analyzing the way in which technical advances make possible new levels of productivity.[5]

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[4]Walters, "Production and Cost Functions", p. 3.

[5]Salter, Technical Change, p. 26.

Brown also agrees that the technology embedded in the production relation acts as a constraint on decision making.[6] His analysis consists of first defining the two types of technological change, 'neutral' and 'non-neutral', and then considering these two categories by analyzing the characteristics of technology in a production function by means of a theoretical abstraction. This "abstract technology" construct of Brown's will be discussed in more depth at a later point in this study.

The mechanisms by which technological constraints are shifted within the production set must also be considered. To accomplish this Nadiri points out that two theories of technical change can be considered.[7] The first and the most common theory in the literature of production functions is the view that changes are of autonomous, and induced nature. This view assumes technical change to be autonomous, neutral and growing at a constant rate and is explained by Nadiri in the following quote:

> ...that the supply of technical change is determined by the state of knowledge and autonomously supplied inventions. Promoters simply discover the commercial use of these techniques. Expectations of future profits determine the adoption rates of new techniques but economic considerations do not determine the nature of the techniques themselves.[8]

The question that arises from this view is whether or not an inherent capital bias can be predicted in technical progress itself or whether a capital bias, for example, is due simply to a substitution effect. The theory of autonomous technical change has been translated into

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[6]M. Brown, <u>On the Theory and Measurement of Technological Change</u>, (hereinafter referred to as Measurement of Technological Change), (Cambridge: Cambridge University Press, 1969), p. 9.

[7]Nadiri, "Theory and Measurement", p. 1146.

[8]Nadiri, "Theory and Measurement", p. 1146.

induced technical change models some of which are explained by Nadiri.

The second view, endogenous theory, states that the level of technical knowledge is not solely determined outside the economic system. They suggest that the level of technology is not only subject to drift over time but that it is also determined, in part, by the amount of resources which are allocated to the production of technology. Researchers into the theoretical aspects of technical change are reviewed by Nadiri.[9]

Whether technological change is produced endogenously or exogenously the effects of such changes are felt only when they have been transmitted into the production process. Once again, at least two theories have been expounded to explain this process. First, technological change can be considered as being embodied in capital. This view is explained in the following excerpt by Salter:

> A... feature of the model is the role of gross investment as the vehicle of technical change. When there is no technical change, investment is required only to make good the depletion of the existing capital stock through physical deterioration, and to add to this stock. But when technical change is taking place, gross investment has another extremely important role: that of providing the necessary specialized capital equipment required for new techniques irrespective of whether or not they are more or less mechanized than their predecessors. Without gross investment, improving technology that requires new capital equipment simply represents a potential for higher productivity; to realize this potential requires gross investment. An economy with a low rate of gross investment is restricted in the rate at which new techniques can be brought into use; an economy with a high rate of gross investment can quickly bring new methods into use, and thus realize the benefits of improving technology.[10]

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[9]Nadiri, "Theory and Measurement", p. 1148.

[10]Salter, Technical Change, p. 63.

Salter has knowingly used this approach as a simplifying assumption in order to remove some of the restrictions imposed by employing the second or "disembodied" view of the transmission of technological change. This simplification is noted by Salter when he states the following:

> ...it is useful to divide the flow of new techniques into two categories: those which require specialized capital equipment, and those which do not. Most new techniques probably fall in the first category.[11]

Thus it can be seen that the disembodied approach includes those technological changes that can be carried into place without the use of specialized capital equipment. Often these take the form of managerial or organizational changes.

Finally, as Nadiri points out, barriers exist for embodied technological change as a mechanism for diffusing technology throughout an industry.[12] The most important barrier is the existing old capital stock. He correctly points out that the replacement of some elements of a plant may serve to reduce costs and prove to be an economic decision when considered in a vacuum but when considered in the context of an entire plant the complementary nature of these elements often precludes replacing them piecemeal.

Production Functions

Activity analysis and the technological change concepts presented in the previous section are very general models. While

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[11] Salter, Technical Change, p. 50.

[12] Nadiri, "Theory and Measurement", p. 1149.

limiting their potential, the application of these theories can be greatly simplified through the artifice of specialized algebraic forms referred to as production functions. In the past, the Cobb-Douglas production function has held a position of prominence in theory dealing with aggregate production functions. Recently its position of prominence has been assumed by what is referred to as the 'Constant Elasticity of Substitution' production function which is popularly known as the CES function. The CES function was independently derived and introduced by two different groups in the early 1960's.[13] In either case these functions have been formulated in an attempt to determine the nature of factor productivity and technical change.

Factor productivity and technical change can tentatively be said to derive from two major determinants: the technical characteristics of the production process and the movement of relative factor prices.[14] Nadiri points out that the technical characteristics of the production process are, the efficiency of production, the bias in technical change, the elasticity of substitution, the scale of operations and the homotheticity of the production function. In order to measure technical change with algebraic production functions, it is necessary to attempt to account for the technical characteristics of the production relationships.

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[13] The CES function was derived independently by two groups, one consisting of Kenneth J. Arrow, Hollis B. Chenery, Bagicha S. Minhas and Robert M. Solow: the other of Murray Brown and John S. de Cani. The first group's work appeared in the Review of Economics and Statistics, Vol. VL, Aug. 1961 and the second group's work appeared in the International Economic Review, Vol. IV, Sept. 1963.

[14] Nadiri, "Theory and Measurement", p. 1141.

In describing production relationships by means of an algebraic function it is also necessary to make certain assumptions about the relationships between inputs and outputs. One simplifying assumption that makes mathematical solutions feasible for the more advanced production functions is constant returns to scale. In describing production functions the term constant returns to scale is the same as the term linear homogeneity. The linearly homogenous case of a production function occurs if a proportionate change in all inputs produces a proportionate change in output. If this relation exists under the conditions of perfect competition then the size of the most efficient unit of production becomes indeterminate.

Returns to scale, one of Nadiri's technical characteristics, is measured by the degree of homogeneity of a function and is often thought of as a time concept. Generally, a short run analysis is concerned with varying one input and is associated with the principles of "diminishing marginal and average returns" for the variable input. Long run analysis is concerned with "returns to scale". In the long run the constraint of installed capital is replaced by a constraint consisting of the fund of technical knowledge available. Brown goes on to define a third period, the secular period,[15] which relaxes the constraint imposed by the fund of technical knowledge. Brown then defines an 'epoch' as being a period through which the fund of technical knowledge remains constant and a secular period as consisting of two or more epochs.

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[15]Brown, Measurement of Technological Change, p. 75.

The elasticity of substitution measures the ease with which factors of production can be substituted for one another within the production process. This can be more precisely stated by saying that the elasticity of substitution measures the percentage change in the proportion of use of the factors as the marginal rate of substitution changes. This measure also tells how rapidly diminishing returns set in to one factor of production when its proportionate use increases.

The efficiency of production techniques is concerned with the technological requirements of the production process and not the effects of relative factor supplies. The efficiency of a new technique is measured by its ability to reduce overall unit costs of factors of production through its application. If the dimensions of the inputs are defined, the efficiency of production can be seen as a scale transformation of inputs into output. On the other hand, a bias in technical change, where the intensity of capital or labour is increased, measures the degree to which a new technique leads to a greater saving in one input than the other. The homotheticity of a production function determines whether the returns to scale are evenly distributed among factors of production.

Before any further diagnosis with technological characteristics is performed a definition of the two general types of technological change should be made. 'Neutral" technological change neither saves nor uses more labour for the same level of output; it is a change which produces a variation in the production efficiency but does not affect the marginal rate of substitution of labour for capital. A

'non-neutral' technological change alters the slope of the production function and can be either labour or capital saving. As an example, if the production function is altered such that the marginal product of capital rises relative to the marginal product of labour for each combination of capital and labour then a capital using technological change has occured. Non-neutral technological changes are produced by variations in the capital intensity of technology and in the ease of substitution of capital for labour.

The Cobb-Douglas Production Function

The Cobb-Douglas production function is represented as follows:

$$X = A\ N^{\alpha}\ C^{\beta}$$

where X = Output or value added
N = Employed labour
C = Utilized capital services
α and β = Elasticities of production for labour and capital respectively
A = Residual due to unspecified factors

This function possesses four primary properties that can be used to explain technical characteristics of the production process. They consist of the following:

1. The parameters α and β represent the elasticities of production for labour and capital respectively.

2. The function is homogenous of the degree $\alpha + \beta$. If $\alpha + \beta$ exceeds unity there are increasing returns to scale and if $\alpha + \beta = 1$ there are constant returns to scale.

3. The marginal product of labour (MPP_L) equals $\alpha(X/N)$ the marginal product of capital (MPP_C) equals $\beta(X/C)$ and the marginal rate of substitution of capital for labour ($MRS_{C\ for\ L}$) equals $\alpha/\beta(C/N)$. As an example the MPP_L declines if labour inputs increase

when $\alpha<1$.

4. One of the main characteristics of the function is that the elasticity of substitution is constant at unity.

The most famous property of the Cobb-Douglas function is that the elasticity of substitution is equal to unity for all combinations of labour and capital and for any degree of returns to scale. For the purpose of this study this is also its major limitation since, because of this property, it is unable to represent a change in the ease of substitution of labour for capital. Imposing the condition of unitary elasticity of substitution also requires the assumption that relative income shares remain constant. Because of the constraints these restrictive assumptions place on any results, the more general CES production function will be used in this study.

The CES Production Function

For a two digit industry[16] the CES function can be stated as follows:

$$X = \gamma[\kappa C^{-\alpha} + (1-\kappa)N^{-\alpha}]^{\nu/\alpha}$$

where X = Output or value added
C = Utilized capital services
N = Employed Labour
γ = Scale parameter denoting efficiency of technology
κ = Degree to which technology is capital intensive
ν = The degree of homogeneity of the function or the degree of returns to scale
$\sigma = \dfrac{1}{1+\alpha}$ = The elasticity of substitution of labour for capital

[16] A two digit industry signifies that there are only two factors of production; in this case labour and capital.

The generality of this function allows for the establishment of technological relationships that would not be possible with a more restrictive function such as the Cobb-Douglas. Brown's concept of an abstract technology[17] can be described by four characteristics all of which are evident in the CES function. This 'abstract technology' clearly involves four of the technical characteristics of the production process previously listed from Nadiri's work and will be somewhat repetetive. Brown's four characteristics are efficiency, economies of scale, capital intensity, and the ease with which capital is substituted for labour.

First, for given inputs and other characteristics of the abstract technology, the efficiency characteristic determines the output that results. This efficiency characteristic can be thought of as a scale transformation of inputs into outputs. For the CES function the parameter 'γ' represents the efficiency of a technology. Previous estimates of γ obtained through using the CES function vary widely and are dependent on the period of fit and on assumptions about the type of embodiement and returns to scale.[18]

The second characteristic of an abstract technology is the degree of homogeneity or the returns to scale. For the CES function this property is characterized by the parameter 'ν' which can assume any value. The CES function can, therefore, represent any degree of returns to scale that an empirical situation dictates. This is an

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[17] Brown, Measurement of Technological Change, p. 13.

[18] Nadiri, "Theory and Measurement", p. 1152.

important attribute for studies concerning a firm since technological progress can alter the way in which inputs are transmitted into outputs. This alteration could be such that a production process, formerly characterized, for example, by decreasing returns is now characterized by constant returns. Previous studies which employed the CES function have found estimates of ν to be greater than unity for time series regressions and about unity for cross section regressions. Unity would indicate constant returns to scale. In any case ν seems to be sensitive both to the utilization rates of inputs and to the level of demand.[19]

The third characteristic of Brown's abstract technology is capital intensity and it is related to Nadiri's characteristic of factor bias in technological change. For the CES function the parameter 'κ' indicates the degree to which the technology is capital intensive. Other authors, such as Ferguson,[20] term this the 'distribution' parameter stating that it is related to relative income distribution. The latter name is more descriptive in that it indicates why the parameter κ is confined to the interval $0<\kappa<1$.[21] This confinement is contrary to the usual definition of capital intensity that proceeds in terms of the quantity of capital relative to the quantity of labour used in the production process. A greater degree of capital intensity can be produced either by a relatively larger amount of capital being supplied to the firm or it can be due to the

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[19] Nadiri, "Theory and Measurement", p. 1152.

[20] C. E. Ferguson, "Time Series Production Functions", The Journal of Political Economy, LVVIII (April 1965), p. 138.

[21] Brown, Measurement of Technological Change, p. 45.

fact that the technology of that firm required a larger amount of capital relative to the amount of labour required. Degrees of capital intensity are reflected in the size of the capital labour ratios, or in this instance the parameter κ, for given relative factor prices.

The marginal rate of substitution of labour for capital can be derived in terms of κ for the CES function:

$$R = \frac{\partial x/\partial C}{\partial x/\partial N} = \frac{\kappa}{1-\kappa}\,(\mu)^{1/\sigma}$$

where μ= Ratio of labour to capital (N/C).

From this relationship R (the marginal rate of substitution of labour for capital) it can be seen that if the production process is highly labour intensive, that is κ is small, then the marginal product of labour is high relative to that of capital for each labour capital ratio. In such a situation any reduction in the labour ratio has to be compensated for through a larger increase in the rate of capital than would have been the case if the production process were less labour intensive. It is in this sense that κ is a measure of the capital intensity of a technology.[22]

Finally, the fourth characteristic used by Brown to describe a technology is the ease with which labour is substituted for capital; the elasiticity of substitution. For the CES function, the elasticity of substitution of labour for capital is defined by the function:

$$\sigma = \frac{1}{1+\alpha}$$

where σ= Elasiticty of substitution.

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[22]Brown, Measurement of Technological Change, p. 48.

If σ is high (i.e. $\sigma>1$), labour is easily substituted for capital and vice versa. The previous definition of R indicated that if labour input is reduced by one unit then capital input must increase proportionately more if factors are not easily substituted one for the other. Thus a low σ indicates a dissimilarity between the factors of production. The implications of σ are as follows. A low σ indicates a dissimilarity between the factors of production and implies diminishing returns to labour will set in more rapidly for an increase in R than is the case when σ is at a higher level. If σ is high, the technology permits easy substitution, an expanding factor can be substituted for one that is held constant and it appears that both factors and output can be expanded indefinitely.

Neutral technological progress is representable in the CES production function through the γ parameter. As Brown indicates:

> for proportionate increases in the parameter γ, produce proportionate increases in output, holding all other things constant. Hence an upward shift in γ represents an upward shift in the efficiency of a CES technology.[23]

Returns to scale are represented by the parameter ν. This parameter determines the degree of returns to scale but it does not indicate how much of any change in output is attributable to the exploitation of economies of scale. Thus, one interpretation of a change in ν can be technological advance. Brown's model produces an estimate of both γ and ν, neither of which affect the marginal rate of substitution of capital for labour. The problems of separating the

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[23] Brown, Theory and Technological Change, p. 54.

causal relations affecting ν are dealt with in the section on limitations.

Non-neutral technological change has been defined by Brown as a change that alters the marginal rate of substitution of labour for capital for each combination of labour and capital.[24] As shown previously this can be expressed as follows:

$$R = \frac{\partial x/\partial C}{\partial x/\partial N} = \kappa' \mu^{1/\sigma}$$

Thus, for the CES production function, non-neutral changes are associated only with variation in κ, the capital intensity parameter, and σ, the elasticity of substitution. In both instances any factor bias present in technological change is measured.

The effect on output of changes in the parameters mentioned above can be shown for the case where capital is growing faster than labour. If the elasticity of substitution, σ, is constant while the capital intensity of the technology increases then the rate of growth of output will increase. If both the parameters σ and κ are rising the rate of growth of output will increase. In the case where labour is growing faster than capital then an increase in κ with σ constant will cause the rate of growth of output to decrease. If both σ and κ are rising while labour is increasing faster than capital the results on output must be determined empirically.

Production functions should satisfy three neo-classical criteria. Brown shows that the CES function satisfies these criteria.[25]

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[24] Brown, Theory of Technological Change, p. 55.

[25] Brown, Theory of Technological Change, p. 46.

First, the marginal products of capital and labour are positive. Second, the marginal product curves of capital and labour slope downwards except when strong economies of scale are present and finally the CES function is able to represent any degree of returns to scale present in an empirical situation. Since it satisfies these criteria the CES function can be used to describe equilibrium conditions and can assume any degree of homogeneity required by an empirical situation.

One of the most significant contributions of the CES function comes from its asymtotic properties. Brown shows[26] that when $\sigma>1$ the CES function possesses no limit. This property has already been discussed in dealing with the elasticity of substitution. However, Brown also shows that when $\sigma<1$ the function reaches a finite maximum as one factor increases while the other is held constant. This result implies diminishing marginal returns for the variable factor. The Cobb-Douglas function does not produce such a finite maximum.

Limitations

Many of the limitations of the CES production function are listed by Brown.[27] First, he notes that the parameter ν combines in one parameter the effect of two forces. Either economies of scale resulting from an expansion of operations or technological change that alters growth could influence ν. In empirical studies

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[26] Brown, Theory of Technological Change, p. 50.

[27] Brown, Theory of Technological Change, p. 59.

both forces affect the homogeneity parameter ν and it would be difficult to distinquish between them. As Brown points out:[28]

> ...the interpretation of the economies of scale measure in empirical situations must be made with caution in any constant elasticity production function.

A second relevant weakness of the CES function in associated with its strength: the specification of an elasticity of substitution which is invariant to changes in factor proportions. For this function, the elasticity of substitution is allowed to change in response to technology but not in response to changes of factor proportions. A specification error could occur and it may be possible to ascribe more to technological change than is due it.

Summary

Production functions are an algebraic simplification of the relationships between the maximum quantities of output and the inputs required to produce this output and between the inputs themselves. In order to achieve plausible results, these functions must satisfy three neo-classical economic criteria. These criteria are satisfied by the CES function in that, first, the marginal products of capital and labour are positive, second, they slope downward except when strong economies of scale are present and, third, it is able to represent any degree of returns to scale that would be required for analyzing an empirical situation. As well as satisfying these criteria the CES function, in reaching a finite maximum when $\sigma<1$

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[28] Brown, Theory of Technological Change, p. 59.

as the variable factor increases, can indicate diminishing returns for the variable factor.

The CES function is capable of expressing technological change through variations in its parameters. Neutral technological progress is expressed in the efficiency parameter γ and the returns to scale parameter ν. Increases in these parameters increases output leaving the marginal rate of substitution unaffected. Non-neutral technological change is reflected in variations in the capital intensity parameter, κ, and the elasticity of substitution parameter, σ. Variations in these parameters produce the following effects. First, a rising capital intensity parameter produces capital using technological change, second, a rise in the elasticity of substitution produces a capital using (saving) change if capital is growing more (less) rapidly than labour. Finally, various effects of the parameters on output can be shown. An increase in the elasticity of substitution, σ, always raises the rate of increase in output. An increase in the capital intensity parameter, κ, raises the rate of growth in output if capital is growing faster than labour. If labour is growing faster than capital and both σ and κ rise the results must be determined empirically.

CHAPTER III

DESIGN OF THE MODEL

Introduction

The task of this chapter is to outline the design of the model that will be used for estimating the parameters of the production function, for Alberta Government Telephones, hereinafter referred to as AGT. This design will be developed and discussed in the following sequence. The first section will discuss the method for developing data and productivity indexes. The second section will present the regression model to be used in estimating the production function. The third section will deal with limitations of the regression model and the last section will be a summary of the objectives of the two stage model.

Productivity Indexes

The only productivity indexes measureable with the statistical techniques available are those ratios devised by dividing outputs by inputs. The number or type of ratios which can be produced by such measurements is equal to the number of inputs measureable since any of these ratios represents only the physical product of the input factor. The methodology to be used in this study for developing productivity indexes using telephone company data was originally presented in a study by Olley at the annual meeting of the Economics

Association of Canada.[1] This study will attempt to replicate portions of the Olley study using data provided by Alberta Government Telephones.

Productivity statistics can be expressed with the following statistical notation:

$$\frac{\Sigma P_0 Q_t - \Sigma p_0 q_t}{\Sigma W_0 F_t}$$

where
- P = Price of gross output
- Q = Quantity of gross output
- p = Price of intermediate inputs
- q = Quantity of intermediate inputs
- W = Weight given to each factor
- F = Quantity of each factor
- o,t = Base and current periods respectively

The numerator of the above function represents a measurement, in some form, of the value added by the firm. Two types of value added can normally be measured. First, 'net value added' can be measured as the total value of sales minus the sum of materials and services purchased by the firm and minus economic depreciation. Economic depreciation is derived through a 'capital census' [the use of statistical life curves to determine remaining plant balances], rather than using the depreciation rates prepared for tax purposes. All values should be expressed in constant dollars. Second, 'gross value added' is measured in the same manner as net value added with the exception that economic depreciation is not deducted. Again, all measurements should be in constant dollars.

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[1] R. E. Olley, Productivity Gains in a Public Utility, (hereinafter referred to as Productivity Gains), (a paper presented to the Annual Meeting of the C. E. A., Winnipeg, Manitoba, 1970).

If the firm is considered in the form of a two digit industry then the denominator of the above equation can be expressed by five general categories. These are shown in Table I and Table II. First, as shown by the tables, capital inputs can be measured as the constant dollar value of capital services only. Capital services should be matched either to gross or net value added.

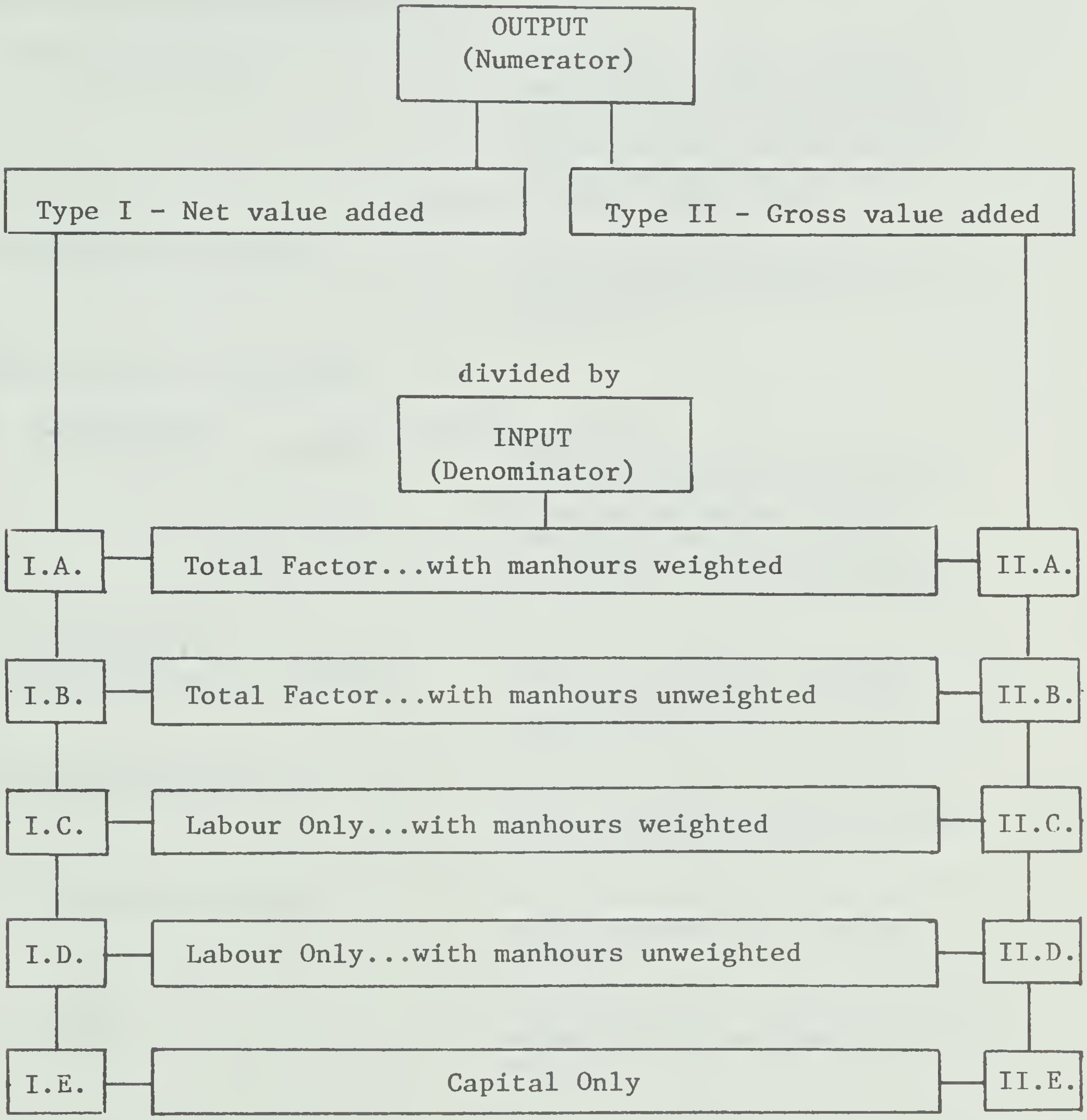
Second, the productivity of labour can be measured by using manhour inputs which are either weighted or unweighted. Unweighted manhours are measured as the constant dollar value of labour only, simply by totalling manhours worked. Weighted manhours are achieved by adjusting the constant dollar value of labour for a change in the quality mix. Such an adjustment is made possible by assuming that changes in average hourly pay reflect changes in quality and that there is no case of serious shortage or over supply of labour.

Finally, total factor productivity is a measurement of the use of all inputs employed in the production process. As with labour, it can be measured in two ways. First, total factor productivity can be measured as the value added, in constant prices, divided by capital service inputs (with or without depreciation) and labour service inputs measured in manhours and adjusted for changing quality mix. Total factor productivity, with manhours unweighted, is measured in the same manner except that the adjustments for quality changes in the labour force are not made.

Since the objectives of this analysis are not as comprehensive as those of Olley's, all of the possible combinations depicted in the tables will not be prepared. A comparison between the movement of

TABLE I

TYPES OF PRODUCTIVITY MEASURES



Source: R. E. Olley, "Productivity Gains", p. 2.

TABLE II

OUTPUT AND INPUT MEASURES

TYPE OF OUTPUT MEASURES	
I Net Value Added:	Total value of sales minus the sum of materials and services purchased from other segments of the economy and minus economic depreciation, all in constant dollar values.
II Gross Value Added:	Same as net value added but with economic depreciation not deducted.
TYPE OF INPUT MEASURES	
A. Total Factor, with manhours weighted:	Value, in constant prices, of capital service inputs matched to output and labour service inputs with the labour input measured in manhours adjusted to reflect changing quality mix.
B. Total Factor, with manhours unweighted:	Same as A but with manhours not adjusted for change in the quality mix.
C. Weighted Manhours:	Constant dollar value of labour only, with adjustment for change in the quality mix.
D. Unweighted Manhours:	Constant dollar value of labour only, with manhours simply totalled.
E. Capital:	Constant dollar value of capital services only, matched to output.

Source: R.E. Olley, "Productivity Gains," p.3.

productivity at AGT and at Bell Canada, the two examples of the industry, can be made using only the 'Type II' index prepared here. The objectives of such a comparison will be to determine whether such movements have the same direction and the same general mangitude. To the extent that productivity measures are prepared for this study they will be employed only in this type of static interpretation of output per unit of resources foregone. No attempt will be made to interpret measures of efficiency or technological change from these productivity indexes.

The Regression Model

At least four different methods have been employed for estimating the parameters of the CES production function. These methods are presented by Nadiri[2] and, in summary, are the following. The first approach is to use a least squares technique to directly estimate the following equation:

$$\log x = \log \gamma + \nu\kappa \log(C/N + \nu \log N + \beta[\log (C/N)]^2 + \nu'_o$$

where, $\beta = \alpha\nu\kappa(1-\kappa)/2$.

The second and most common approach is a stepwise procedure that estimates the parameters κ and α from the side relation of the ratio of marginal productivity. Then, using these estimates, $\hat{\kappa}$ and $\hat{\alpha}$, the two remaining parameters, ν and γ, are estimated. The shortcomings of this procedure are that the;

...assumption that marginal productivity relations

[2]Nadiri, "Theory and Measurement", p. 1154.

> hold which in turn requires that the returns to scale parameter is unity. It also assumes that the responses of κ and α to changes in relative prices, w/g, are identical, that the marginal productivity conditions hold instantaneously, and that the current relative prices are exogenous and good proxies for their expected values.[3]

W/g represents the ratio of returns of labour to the returns of capital. The best estimates of σ can be found by regressing log (w/g) on log (C/N).

The third approach, as proposed by Bodkin and Klein, is a non-linear maximum likelihood technique. This method is explained by Brown in the following quotation:

> ...the model is linearized by a first-order Taylor series approximation about the initial guesses of the parameters. The usual normal equations are derived and corrections of the parameter estimates are calculated by iteration until convergence criteria are satisfied. ...; the Taylor series approximation - sometimes called the Gauss-Newton method - may not converge.[4]

Finally, a Bayesian estimation technique has been developed. This technique estimates the parameters of an average production function and not the efficient combination of inputs suggested by economic theory.

The two techniques to be used in this study will be explained in more detail. For the indirect linear estimation technique Brown[5] shows that given the logarithmic transform of the CES production function, $\log x = \log \gamma - \nu/\alpha \log (\kappa C^{-\alpha} + (1-\kappa)N^{-\alpha})$

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[3]Nadiri, "Theory and Measurement," p. 1155.

[4]Brown, Theory and Measurement, p. 134.

[5]Brown, Theory of Technological Change, p. 129.

where, X = Output
C = Capital Services
N = Labour inputs
γ = Technological efficiency parameter
κ = Capital intensity parameter
ν = Degree of returns to scale
$\alpha = -1-(1/\sigma)$
σ = Elasticity of substitution

it is not possible to obtain estimates of the parameters directly. Estimates of κ and α, must therefore be obtained by a 'side relation'. This side relation is the CES expansion path function and can best be fitted to data by the method of least squares. It represents a logarithmic transformation of the equation introduced in the previous chapter for the marginal rate of substitution. This function can be given as follows;

$$\frac{\kappa}{1-\kappa}(u)^{1/\sigma}$$

where, $u = N/C$

For the present study, since the data on relative factor prices are at the level of the firm it is more likely that relative factor inputs are influenced by relative factor prices and simultaneous equation bias should be minimized.[6] To obtain the estimates of κ and α, Brown proposed the use of the side relation in the following form;

$$\log N/C = \sigma(1-\lambda) \log \kappa' + \sigma \log q/w + \lambda \log (N/C)_{-1}$$

where q/w = Ratio of rental value of capital to the wage rate of labour
$\kappa' = \kappa/(1-\kappa)$
λ = Rigidity parameter

The estimates produced by this side relation, $\hat{\kappa}$ and $\hat{\alpha}$, can then be inserted into the following function to determine a new

[6] Brown, Theory of Technological Change, p. 130.

variable,

$$\hat{V} = [\hat{\kappa}C^{-\alpha} + (1-\hat{\kappa})N^{-\hat{\alpha}}].$$

Having obtained the estimate for V, the following relation can then be fitted to data:

$$\log x = \log \gamma - \nu/\alpha \log \hat{V}.$$

Brown also points out that it may be necessary to include a trend term which consists of adding the expression αt to the previous equation. The estimate of α represents the rate at which the capital intensity parameter κ changes. Fitting data to the CES production function allows the isolation of a technological epoch, a period during which no non-neutral technological change occurs. Technological epochs can be isolated with the techniques described by Brown[7] and which are only briefly outlined below.

The statistical procedure for isolating epochs consists of fitting a CES function to a time period, deriving a set of parameter estimates, fitting the same CES function to a contiguous time period and deriving a second set of parameter estimates. A third fit is then obtained from the combined observations for the two time periods. An F test can then be used to determine if a structural break has occurred between the two time periods. If a structural break is uncovered an epoch has been isolated and the procedure is repeated beginning with the observations that occur just after the structural break.

Once significant epochal estimates of the parameters of the

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[7]Brown, Theory of Technological Change, p. 114.

CES function are obtained it will be possible to measure the three forces affecting total changes in output. The three forces consist of changes in inputs, neutral technological change and non-neutral change. The finite differencing method employed by Brown[8] can then be employed, using the epochal estimates of the CES parameters, to yield a measure of the three forces affecting the total change in output.

The non-linear maximum likelihood technique can be outlined as follows;

Let $r_o = \nu_o/\alpha_o$

and $G = [\kappa_o C^{-\alpha_o} + (1-\kappa_o)N^{-\alpha_o}]$

where zero subscripts indicate initial values. The first order Taylor series approximation can be written as;

$$[x] = f_o + \frac{\partial x}{\partial \gamma}(\gamma - \gamma_o) + \frac{\partial x}{\partial \kappa}(\kappa-\kappa_o) + \frac{\partial x}{\partial \alpha}(\alpha-\alpha_o) + \frac{\partial x}{\partial r}(r-r_o)$$

$$= f_o + G^{-r_o}(\gamma-\gamma_o) + r_o\gamma_o G^{-r_o-1}[C^{-\alpha_o} - N^{-\alpha_o}](\kappa-\kappa_o)$$

$$- r_o\gamma_o G^{-r_o-1}[\kappa C^{-\alpha_o}\log C + (1-\kappa_o)N^{-\alpha_o}\log N(\alpha-\alpha_o)$$

$$+ \gamma_o G^{-r_o}\log G(r-r_o)$$

where [ ] = predictions of the linearized model and

f_o = original function evaluated at initial guesses.

The function Z is now formed;

$$Z = \Sigma[x_i - [x_i]]^2$$

where i = an observation index

and the elements of the correction vector, $(\gamma-\gamma_o)$, $(\kappa-\kappa_o)$, $(\alpha-\alpha_o)$ and $(r-r_o)$,

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[8] Brown, Theory of Technological Change, p. 118.

enter linearly in Z and can be found by application of least squares. This iterative process repeats itself until a convergence criteria is satisfied. Provided that the original estimates are in the neighbourhood of the true parameters this method can yield estimates of the CES parameters directly.

Limitations

There are a number of limitations to using this method for estimating the CES function not the least of which is the considerable burden of fitting this function to the data. The use of the CES function with its underlying generality and Brown's approach to technological change have been specified a priori with no experimentation. The base period employed is assumed to contain no non-neutral technological change. Technological changes of a neutral and non-neutral nature have been assumed to occur at the same time and it is also probable that the statistical techniques employed will not eliminate the possibility of sub-epochs existing within any epochs measured. The model does not explicitly account for a change in the demand mix so that changes in taste not associated with changes in technology can be separated from changes due to technological change. Finally, it should be mentioned that there are data deficiencies as well as all the problems associated with the use of aggregate time series data. Problems of data deficiencies will be dealt with in the succeeding chapter.

Summary

In choosing the model to be used in measuring productivity movements and estimating a production function, the intent was to provide a model that could be used for decision making by the firm. By using the CES function there are some major advantages to be gained as oppossed to using a function such as the Cobb-Douglas. Most important, it can represent all of the technological characteristics of the Cobb-Douglas as well as measure a change in the elasticity of substitution.

The generality of the CES function combined with Brown's 'epoch' approach allows the measurement of technological change. In so doing it provides considerable information to the firm with respect to its technological processes. The comparison of productivity indexes should determine if AGT is representative of the telephone industry. If this is found to be true, some tentative generalizations about the sources of productivity for the telecommunications industry can be made.

One word of caution must be made. While the power of the CES function is derived from its generality the cost of this generality is the burden of fitting this function. These problems, along with the problems always existing with the use of time series data, must be overcome before any value can be gained from this approach.

CHAPTER IV

DATA AND PRODUCTIVITY INDEXES

Introduction

As it exists today AGT is a provincially owned corporation serving the communication needs of Albertans. Just as Olley's study exhibited for Bell Canada, AGT has also undergone rapid growth over the last twenty years and has been transformed from a labour intensive to a capital intensive organization. For part of the time period under observation, 1950 to 1958, AGT was a department of the provincial government and was converted to crown corporation status only in the latter year. The purpose of this study is to explain the movements of productivity at AGT, to examine their similarity to those shown by Bell Canada productivity indexes and finally, with the use of an aggregate production function, to determine the source of any productivity gains or losses.

In a communications company it would be possible to increase output while supplying a lower "grade of service" using the same amount of labour and capital inputs. The result would be higher productivity indexes. However, in this instance measures of service indicate that service has not deteriorated but has shown continuous improvement throughout the period.

An F test will be used in comparing the two sets of productivity indexes. It will test for variability between the two processes.

The null hypothesis will be that there is essentially no difference between the variances of the two populations.

The organization of this chapter will proceed in the following order. The first section will present the method of measuring "value added" or output, the second section the method of measuring labour inputs, and the third section the method of measuring capital inputs. The fourth section will present the indexes prepared with AGT data. The fifth section will compare these productivity schedules with those prepared by Olley. The sixth section will discuss the limitations and the final section will present conclusions.

Measurement of Value Added

AGT gross revenues are derived from a variety of sources by means of selling, with monopoly privileges, a set of diverse communications services to the public. For this study gross revenues as reported by the company have been supplemented through the addition of uncollectibles (those services supplied for which no rent was collected). Uncollectibles represent approximately 0.5 per cent of the total. For this study gross revenues were divided into the following categories:

A. Non-Toll Categories

1. Local service revenues
2. Directory and advertising revenues
3. Miscellaneous service revenues

B. Message and Other Toll Categories

1. Trans-Canada message toll

2. USA and overseas message toll

3. Adjacent member message toll

4. Intra-AGT message toll

5. Other toll revenues

A number of problems were encountered while attempting to gather the necessary data for preparing a "value added" schedule for the twenty year period. First, for purposes of financial accounting the year end was changed from March 31 to December 31 during the fiscal year 1966. All data prior to 1967 was adjusted to a calendar year basis so that the results could be compared with those of Olley's study.[1] Second, total toll revenue statistics were available for the full twenty year period but the breakdown into five homogenous sub-categories was not available. These 'distributed' revenue statistics for toll categories were available only for periods in the 1960's thus requiring the distribution of toll revenues into sub-categories for the earlier periods to be estimated by means of multiple regression. Year end current dollar revenue figures are shown by categories in Table III columns 2 through 9 for the years 1950 to 1969.

Because this study is more closely concerned with technological sources of productivity gains and, as Salter points out[2], gross investment is the major vehicle of technical change, the revenue, capital and total productivity indexes used in this study will be

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[1] These year end adjustments were made by means of 7 month and 12 month revenue statements supplied by AGT's General Accounting Department.

[2] Salter, Technical Change, p. 63.

related to gross value added. Since one of the productivity index series used by Olley is developed using gross value added statistics these indexes can be compared.

To derive 'gross value added' gross revenues must be reduced by the value of intermediate goods and services purchased by the firm and by the value of indirect taxes paid by the firm. Statistics on intermediate goods and services are presented in Table III, column 11. Because AGT is a crown corporation there are no indirect taxes to be subtracted. Purchases used in construction are capitalized and are therefore not subtracted. Column 12 of Table III is then derived by subtracting columns 11 from 10 (the total of 2 through 9) and represents gross value added in current prices.

To adjust current dollar figures of Table III into constant dollar figures, price indices of the Laspeyres variety were employed. For most categories of revenues, indexes supplied by the telephone industry were used.[3] These indices were all converted to employ 1967 as the base year. The only exception to this source was for miscellaneous revenues where the DBS[4] 'implicit price index for gross national expenditures' was used. AGT's aggregate price level change (a composite of the individual indexes) is shown as the graph entitled exhibit I.

The constant dollar value of gross revenues and the constant dollar value for gross value added (along with the sub-categories)

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[3]Since these indexes are of a proprietary nature their explicit source cannot be divulged nor can they be published directly.

[4]Dominion Bureau of Statistics, Prices and Price Indexes Vols. 27-47 (Ottawa: Queen Printer), 1950 - 1969.

TABLE III

TOTAL REVENUES IN CURRENT DOLLARS

Year	Non-Toll Revenues			Message and Other Toll Revenues					Total Revenues	Interm. Goods and Services	Total Value Added
	Local Service	Directory	Miscell--aneous	Trans-Canada	USA and Overseas	Adj. Memb.	Others	Intra-AGT			
Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8	Col. 9	Col. 10	Col. 11	Col. 1
1950	2,967	108	275	770	403	60	56	3,007	7,650	147	7,502
1951	3,368	117	391	877	459	68	64	3,425	8,773	167	8,605
1952	3,851	128	446	1,040	545	81	76	4,062	10,232	231	10,001
1953	4,439	152	566	1,206	632	94	88	4,712	11,893	266	11,627
1954	5,080	197	654	1,349	707	105	99	5,268	13,461	316	13,144
1955	5,714	217	941	1,541	807	120	113	6,018	15,475	415	15,059
1956	7,095	242	672	1,832	960	143	134	7,154	18,236	552	17,684
1957	8,122	268	522	1,983	1,039	155	145	7,743	19,980	567	19,412
1958	9,079	299	203	2,297	1,204	180	168	8,970	22,403	768	21,634
1959	10,303	419	50	2,723	1,427	213	200	10,631	25,968	693	25,275
1960	11,353	550	48	3,130	1,640	245	230	12,222	29,421	933	28,487
1961	12,511	608	155	3,532	1,851	276	259	13,791	32,986	1,113	31,873
1962	13,743	681	1,049	3,847	2,016	301	282	15,022	36,945	1,194	35,751
1963	15,068	870	1,488	4,219	2,211	330	310	16,475	40,975	1,202	39,772
1964	16,249	994	1,896	4,907	2,571	384	360	19,160	46,526	1,150	45,376
1965	17,228	1,131	1,716	5,812	3,046	455	427	22,693	52,511	1,631	50,880
1966	19,275	1,202	1,719	6,962	3,649	545	511	27,185	61,052	2,078	58,973
1967	24,878	1,501	2,327	8,463	4,435	663	622	33,046	75,940	2,082	73,857
1968	28,858	1,879	2,364	9,708	5,087	760	713	37,905	87,276	3,043	84,223
1969	32,149	1,986	2,430	11,265	5,903	882	828	43,984	99,429	3,433	95,996

Notes: 1. All figures are represented in current dollar values.
2. All figures are expressed in thousands. (000)

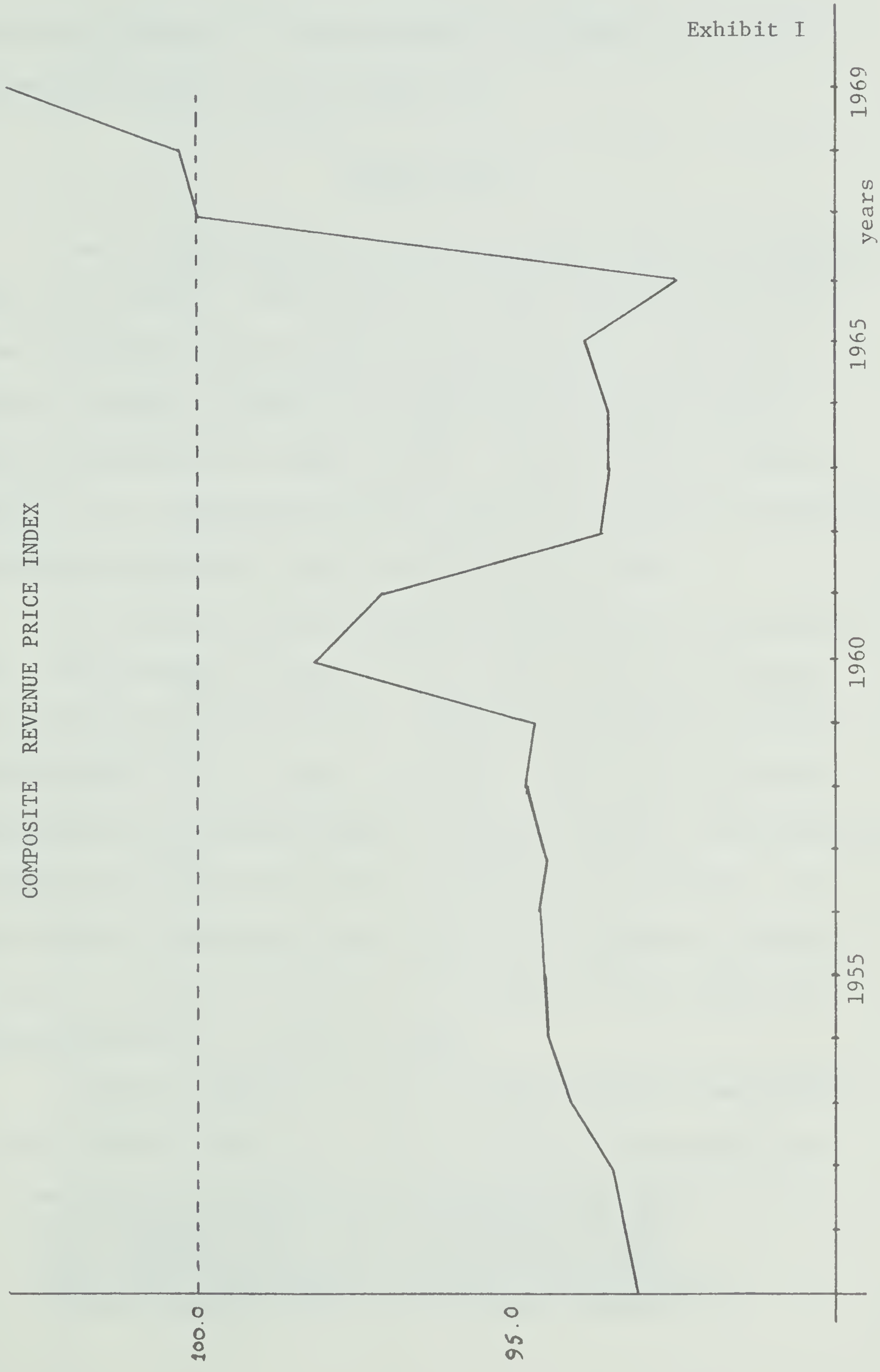
Exhibit I
COMPOSITE REVENUE PRICE INDEX
100.0
95.0
1955
1960
1965
1969
years

are shown in Table IV. The categories and columns of this table correspond exactly to those of Table III.

Labour Inputs

Labour inputs are one of the two primary factors of production used in this study. The statistics on labour inputs are generated from the monthly payroll reports obtained from AGT. Total wages paid per category per year and average number of employees per year were obtained from these records. Because there were numerous changes in the categories of labour involved, the number of categories for any one year varied from 14 in 1950 to 29 in 1969.

The total for manhours worked is derived from total manhours paid. An adjustment was made for sick leave, vacations, holidays and overtime bonus for each year. Manhours paid were also reduced by the amount of wage expense capitalized. This latter adjustment was achieved by applying estimates, supplied by AGT, of the proportion of wages to be capitalized and the proportion to be expensed. Manhours worked, calculated in the above manner, are shown in column 2 of Table V. Column 3 of Table V is the value, in 1967 wage rates, including benefits, of manhours worked.

Quality weighting of labour is achieved by the same methods as used by Olley. These are outlined in the following quotation:

> Weithting for quality was achieved by giving each hour worked in each of the 28 categories in each year, a weight equal to the ratio of the average total hourly remuneration (including fringe benefits) of that category in 1967 to the average remuneration

TABLE IV

TOTAL REVENUES IN CONSTANT DOLLARS

Year	Non-Toll Revenues			Message and Other Toll Revenues					Total Revenues	Inter. Services	Total Value Added
	Local Service	Dir--ectory	Miscel--laneous	Trans-Canada	USA and Overseas	Adj. Mem.	Others	Intra-AGT			
ol.1.	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8	Col. 9	Col. 10	Col. 11	Col. 12
1950	4,127	160	453	719	424	54	52	2,837	8,832	243	8,588
1951	4,685	174	581	812	484	61	59	3,231	10,090	249	9,841
1952	5,357	188	633	950	573	71	69	3,833	11,677	328	11,349
1953	6,175	225	798	1,072	661	78	78	4,445	13,536	376	13,160
1954	7,066	292	901	1,169	744	84	85	4,969	15,315	436	14,879
1955	7,949	298	1,275	1,345	856	98	98	5,672	17,596	563	17,032
1956	9,870	325	877	1,606	1,019	116	118	6,793	20,678	721	19,957
1957	11,297	346	661	1,741	1,136	127	128	7,304	22,745	719	22,025
1958	12,629	374	253	2,027	1,315	147	149	8,361	25,257	955	24,301
1959	14,332	484	60	2,393	1,561	177	175	9,387	28,572	840	27,732
1960	15,792	613	58	2,779	1,640	208	204	10,797	32,094	1,114	30,980
1961	17,404	672	184	3,227	1,818	243	237	12,337	36,125	1,320	34,805
1962	19,117	743	1,224	3,637	1,972	278	267	14,403	41,645	1,393	40,251
1963	20,960	870	1,709	4,067	2,157	314	299	15,796	46,175	1,381	44,793
1964	22,603	994	2,123	4,746	2,520	368	349	18,363	52,069	1,287	50,781
1965	23,590	1,131	1,865	5,707	2,973	442	419	21,760	57,891	1,772	56,119
1966	26,394	1,202	1,786	6,927	3,566	543	509	26,890	67,820	2,160	65,659
1967	24,878	1,501	2,327	8,463	4,453	663	622	33,046	75,940	2,082	73,857
1968	28,858	1,879	2,287	9,717	5,092	760	714	38,482	87,793	2,944	84,848
1969	32,149	1,986	2,245	11,299	5,654	887	830	41,612	96,665	3,172	93,492

Notes: 1. All figures are represented in constant dollar values.
2. All figures are expressed in thousands (000).

for all hours worked by all employees in 1967.[5] The assumption of this method is that quality is reflected in the proportionate pay rates. This assumption is only true if there were no labour shortages or oversupplies existing in the labour market during the year. Such a quality measure accounts only for changes in the labour mix and does not account for general changes such as education. The total manhours worked adjusted for quality changes on this basis are presented in column 4 of Table V and their value, in terms of 1967 average wage rates, are presented in column 5 of Table V. Limitations of this method will be discussed in a later chapter.

Capital Inputs

Capital services are the second factor of production for this study. The statistics for gross capital value and for capital services are presented in Table VI. Column 2 is total gross capital investment per year and column 3 is capital services per year. The use of gross capital investment and gross value added as the denominator and numerator for the productivity index respectively, contains an inconsistency in a national accounting sense. The numerator contains an output item, plant written off to retirement, that cannot be considered as other than an intermediate input that is being resold.

Current dollar gross capital values were derived from age distributed capital accounts. There were 46 accounts involved and

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[5] Olley, "Productivity Gains", p. 14.

TABLE V

LABOUR INPUTS

Year	Unadjusted Labour Inputs		Labour Inputs Adjusted for Quality	
	Manhours Worked	Value in 1967 Dollars	Manhours Worked	Value in 1967 Dollars
1950	2,762	8,619	1,992	6,217
1951	3,247	10,131	2,376	7,414
1952	3,815	11,903	2,832	8,837
1953	4,141	12,921	3,100	9,673
1954	4,239	13,226	3,262	10,179
1955	4,342	13,548	3,412	10,648
1956	4,870	15,196	3,904	12,180
1957	6,031	18,818	4,727	14,750
1958	6,442	20,101	5,186	16,181
1959	6,600	20,594	5,445	16,990
1960	6,239	19,446	5,243	16,359
1961	6,349	19,808	5,400	16,849
1962	6,683	20,853	5,778	18,027
1963	7,387	23,049	6,541	20,410
1964	7,754	24,194	6,893	21,506
1965	8,297	25,889	7,440	23,215
1966	9,137	28,507	8,355	26,070
1967	10,062	31,395	9,426	29,410
1968	9,807	30,597	9,445	29,470
1969	9,393	29,308	8,950	27,924

these have been accumulated to form 6 homogenous categories which are: Outside Plant, Station Equipment, Buildings, Central Office Equipment, Vehicles and Tools. For each account the age distributed capital for each transaction year was deflated using a price index. With the exceptions of Buildings, Vehicles and Tools accounts the price indexes were supplied by the telephone industry.[6] For the former three accounts price indexes were derived from those published by the Dominion Bureau of Statistics.

Once having derived constant gross capital values, capital services are derived by the following formula:

$$\text{Capital Services} = \left[\frac{\text{Gross Value Added in 1967 - Labour Value in 1967}}{\text{1967 Value of Gross Stocks of Physical Capital}}\right] \times \left[\begin{array}{l}\text{Gross Capital}\\ \text{Value/year}\end{array}\right]$$

This method has the characteristic of exactly distributing output in the base year between the factors of production.

Productivity Indexes

Having obtained the data for gross value added, for inputs in the form of capital services and for inputs of labour in the form of manhours paid, adjusted and unadjusted for quality, there are four productivity index series which can be prepared. These are a partial index of capital, partial indices for unweighted labour and for labour weighted for quality and total factor productivity using either one or the other methods of measuring labour. The value of labour

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[6] These indexes are of a proprietary nature and for this reason will not be reproduced in this study nor will current dollar capital values be given.

TABLE VI

GROSS CAPITAL STOCK

Year	Outside Plant	Station Equipment	Buildings	Central Office Equipment	Vehicles	Tools	Total Gross Capital	Capital Services
Col.1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8	Col. 9
1950	40,106	4,098	4,823	7,995	213	145	57,343	5,924
1951	42,077	4,861	6,333	9,944	254	159	63,631	6,573
1952	44,057	5,725	7,743	13,465	291	182	71,465	7.383
1953	47,611	6,784	8,624	16,873	343	206	80,442	8,310
1954	51,385	7,869	8,969	19,614	408	229	88,475	9,140
1955	59,308	9,448	11,374	22,964	484	262	103,843	10,728
1956	63,008	11,177	12,950	29,751	618	290	117,799	12,169
1957	67,472	13,253	21,887	40,381	698	392	144,086	14,885
1958	70,937	14,953	23,736	53,414	828	481	164,350	16,979
1959	75,095	17,700	25,674	59,167	1,034	630	179,302	18,523
1960	78,581	20,233	29,325	70,487	1,214	812	200,656	20,729
1961	82,062	22,668	31,035	82,098	1,351	1,048	220,264	22,755
1962	85,459	25,988	32,980	90,873	1,557	1,162	238,022	24,590
1963	90,061	30,115	34,573	102,290	1,761	1,240	260,043	26,865
1964	95,603	35,055	37,901	115,146	2,254	1,498	287,449	29,696
1965	103,295	40,971	42,239	134,456	2,693	1,753	325,409	33,618
1966	116,608	49,016	46,912	152,007	3,376	1,872	369,794	38,203
1967	126,971	60,079	50,532	168,415	3,673	1,553	411,225	42,483
1968	138,955	69,452	53,246	183,650	3,812	1,918	451,036	46,496
1969	148,220	78,105	55,370	199,491	4,015	1,991	487,195	50,332

and capital inputs and of gross value added in terms of constant dollars are summarized in Table VII. This study has used gross capital investment in measuring capital services and all productivity indexes will be related to this factor. All indexes are prepared according to the formula developed in Chapter III.

For the years 1950 to 1969 Table VIII columns 2 through 6 show the respective productivity indexes. Column 2 is the partial index of capital productivity, columns 3 and 4 are the partial indexes of labour, unweighted and weighted, and columns 5 and 6 are the total factor productivity indexes with labour unweighted and weighted for quality. The corresponding columns in Table IX present the annual percentage gains in productivity for the factors of production.

From Tables VIII and IX some important trends can be seen. First, labour productivity with manhours unweighted rose from an index of 42.35 in 1950 to an index of 135.59 in 1969 (1967 is the base year). This change represents a cumulative percentage gain of 124.63 per cent for the years 1950 through 1969 and of 96.10 per cent for the years of 1952 to 1967. Labour productivity with manhours weighted for quality changes rose from an index of 55.00 to 133.33 by 1969. This represents a cumulative percentage gain of 94.67 per cent for the years 1950 through 1969 and of 70.97 per cent for the years 1952 through 1967. Unweighted manhours is a higher index because it includes productivity gains which were achieved by using higher quality labour. The effects of the quality change are removed in the weighted index series.

The increase in capital productivity was not nearly so great

TABLE VII

SUMMARY OF INPUTS AND OUTPUTS

Year	Capital	Labour Services		Total Factor Inputs		Gross Value
		Unweighted	Weighted	Labour Unweighted	Labour Weighted	
1950	5,924	8,619	6,217	14,543	12,141	8,588
1951	6,513	10,131	7,414	16,645	13,928	9,841
1952	7,383	11,903	8,837	19,286	16,220	11,349
1953	8,310	12,921	9,673	21,231	17,983	13,160
1954	9,140	13,226	10,179	22,367	19,320	14,879
1955	10,728	13,548	10,648	24,276	21,376	17,032
1956	12,169	15,196	12,180	27,366	24,350	19,957
1957	14,885	18,818	14,750	33,704	29,635	22,025
1958	16,979	20,101	16,181	37,080	33,160	24,301
1959	18,523	20,594	16,990	39,118	35,514	27,732
1960	20,729	19,466	16,359	40,196	37,089	30,980
1961	22,755	19,808	16,849	42,564	39,604	34,805
1962	24,590	20,853	18,027	45,443	42,617	40,251
1963	26,865	23,049	20,410	49,914	47,275	44,793
1964	29,696	24,194	21,506	53,890	51,202	50,781
1965	33,618	25,889	23,215	59,507	56,833	56,119
1966	38,203	28,507	26,070	66,711	64,273	65,659
1967	42,483	31,395	29,410	73,878	71,894	73,857
1968	46,396	30,597	29,470	76,994	75,866	84,848
1969	50,332	29,308	27,924	79,641	78,256	93,492

TABLE VIII

PRODUCTIVITY RESULTS

Year	Capital Produc- -tivity	Labour Productivity		Total Productivity	
		Unweighted	Weighted	With Labour Unweighted	With Labour Weighted
1950	83.39	42.35	55.00	59.06	68.86
1951	86.11	41.28	52.85	59.12	68.78
1952	88.42	40.52	51.24	58.85	68.11
1953	91.09	43.28	54.20	61.98	71.23
1954	93.64	47.79	58.22	66.52	74.97
1955	91.33	53.44	63.72	70.16	77.56
1956	94.33	55.82	65.23	72.93	79.78
1957	85.11	49.74	59.46	65.35	72.35
1958	82.33	51.36	59.82	65.54	71.34
1959	86.12	57.22	64.99	70.89	76.01
1960	85.96	67.23	75.43	77.07	81.31
1961	87.98	74.70	82.28	81.77	85.54
1962	94.15	82.06	88.93	88.57	91.94
1963	95.91	82.23	87.42	89.74	92.93
1964	98.36	89.20	94.03	94.23	96.54
1965	96.02	92.13	96.26	94.31	96.12
1966	98.86	97.92	100.32	98.42	99.44
1967	100.00	100.00	100.00	100.00	100.00
1968	104.74	117.90	114.60	110.20	108.29
1969	106.84	135.59	133.33	117.39	116.29

TABLE IX

%GAIN OF PRODUCTIVITY RESULTS

Year	Capital Produc--tivity Gains	Labour Productivity Gains		Total Productivity Gains	
		Unweighted	Weighted	With Labour Unweighted	With Labour Weighted
1951	+3.26	-2.53	-3.91	+0.12	-0.12
1952	+2.68	-1.84	-3.24	-0.47	-0.97
1953	+3.02	+6.81	+5.98	+5.33	+4.57
1954	+2.80	+10.42	+7.42	+7.32	+5.24
1955	-2.47	+11.82	+9.45	+5.47	+3.46
1956	+3.28	+4.45	+2.37	+3.94	+2.86
1957	-9.77	-10.89	-8.85	-10.39	-9.31
1958	-3.27	+3.26	+0.61	+0.29	-1.39
1959	+4.60	+11.41	+8.64	+8.17	+6.55
1960	-0.19	+17.49	+16.06	+8.72	-6.96
1961	+2.35	+11.11	+9.08	+6.10	+5.21
1962	+7.01	+9.85	+8.08	+8.32	+7.47
1963	+1.87	+0.21	-1.70	+1.32	+0.32
1964	+2.55	+8.48	+7.56	+5.00	+4.67
1965	-2.38	+3.28	+2.37	+0.08	-0.43
1966	+2.96	+6.28	+4.22	+4.37	+3.46
1967	+1.15	+2.12	-0.32	+1.60	+0.56
1968	+4.74	+17.90	+14.66	+10.20	+8.86
1969	+2.00	+15.00	+16.28	+6.53	+6.82
CUMULATIVE PER CENTAGE GAIN					
1950-69	+26.25	+124.63	+94.76	+72.00	+54.82
1952-67	+13.56	+96.10	+70.97	+55.63	+40.22

as that for labour productivity though it rose steadily from 83.39 in 1950 to 106.84 in 1969. For the years 1950 to 1969 the cumulative percentage gain was 26.25 per cent and for the period 1952 to 1967 it was 13.56 per cent. Unlike labour productivity there is no significant upturn during the latter part of the period and the partial index of capital shows a rather steady but slow growth.

Total factor productivity grew from 59.06 in 1950 to 117.39 in 1969 with labour unweighted and from 68.86 to 116.29 with labour weighted for a change in the quality. The growth rate in total factor productivity exhibited over the period is steady.

The impact of growth in total productivity can be summarized by measuring cumulative percentage gains for the period. The most conservative estimate of total factor productivity is 54.82 per cent or 2.74 per cent per year, for labour productivity 94.67 per cent or 4.74 per cent per year and for capital productivity 26.25 per cent or 1.31 per cent per year. The Economic Council of Canada cites a goal of approximately 2 per cent for total factor productivity for the nation. The relatively high growth rate for labour is offset by a relatively low growth rate for capital.

There are some observations for which no explanation can be offerred. There was an across the board decrease in productivity in 1957. This period does correspond to AGT's transformation to a crown corporation but this incident is not offerred as an explanation of the resulting productivity losses. In addition, the period 1958 to 1960 can be marked as the period during which capital became the dominant input. Also, labour productivity for the years 1968 to

1969 shows a significant departure in the rate of growth from the previous years and this change is also reflected in the total factor productivity indexes.[7]

Comparisons

A cursory comparison of the above indexes to those of Olley's using Bell Canada data reflect some similarities and differences. The Bell Canada indexes prepared by Olley are presented in Table X and the corresponding percent gains in productivity per year are presented in Table XI. The cumulative percent gain for unweighted labour for the entire period at Bell Canada was 122.40 and for weighted labour it was 111.40. This compares to AGT percent gains over this period (1952 through 1967) of 96.10 per cent and 70.97 per cent respectively. The Bell Canada indexes for capital productivity show an average annual loss of -.9 per cent while AGT's exhibit a slow but steady growth. Moreover Bell Canada capital productivity indexes show a significant turn around from decreasing to increasing productivity in about 1961. This phenomenon does not exist at AGT. Finally the series on total factor productivity cover the same general range for both Bell and AGT, particularly when labour is considered in its unweighted form, but the AGT series exhibit a much steadier growth than that of Bell Canada's. This can be seen as a result of the divergence between the partial productivity

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[7]This decrease can be attributed to a series of variables. First, there was an absolute decrease in the number of employees over this period. Second,problems in categorizing labour may have led to greater capitalization of wages for this period. Third, the method of constructing constant dollar labour values produces an upward bias prior to the base year and downward bias subsequent to it.

TABLE X

BELL CANADA INDEXES OF TOTAL FACTOR PRODUCTIVITY BASED ON GROSS VALUE ADDED AND GROSS PLANT WITH WEIGHTED AND UNWEIGHTED MANHOURS; INDEXES OF OUTPUT PER UNIT OF WEIGHTED AND UNWEIGHTED LABOUR INPUT, AND OF CAPITAL INPUT, 1952 TO 1967

(1967 = 100)

Year	Total Factor Productivity (Gross Plant Combined With)		Output per Unit of Labour Input		Output per Unit of Capital Input
	Weighted Manhours	Unweighted Manhours	Weighted Manhours	Unweighted Manhours	
1952	61.9	58.0	34.8	31.3	102.8
1953	64.4	60.9	37.1	33.8	102.5
1954	64.9	61.2	38.2	34.5	100.3
1955	64.6	60.8	38.8	34.9	98.0
1956	64.9	61.2	39.6	35.6	96.1
1957	68.1	64.3	43.1	38.7	96.8
1958	69.0	66.0	46.1	42.4	92.2
1959	72.1	70.5	51.3	49.0	90.7
1960	73.9	73.2	56.0	54.9	88.4
1961	77.2	77.4	62.5	62.7	87.8
1962	82.3	82.5	69.7	70.1	90.7
1963	83.1	83.1	72.6	72.6	89.7
1964	86.6	86.7	77.8	78.1	91.8
1965	89.8	90.0	82.8	83.2	93.9
1966	93.8	93.4	88.6	87.1	96.8
1967	100.0	100.0	100.0	100.0	100.0

Source: R. E. Olley, "Productivity Gains", p. 30.

TABLE XI

BELL CANADA YEAR-TO-YEAR AND AVERAGE ANNUAL PERCENTAGE CHANGES IN OUTPUT PER UNIT OF FACTOR INPUT, BASED ON GROSS VALUE ADDED AND GROSS PLANT, 1952 TO 1967

YEAR	TOTAL FACTOR PRODUCTIVITY (Gross Plant Combined With)		OUTPUT PER UNIT OF LABOUR INPUT		OUTPUT PER UNIT OF CAPITAL INPUT
	Weighted Manhours	Unweighted Manhours	Weighted Manhours	Unweighted Manhours	
1953	4.0	5.0	6.6	8.0	-0.3
1954	0.8	0.5	3.0	2.1	-2.1
1955	-0.5	-0.7	1.6	1.2	-2.3
1956	0.5	0.7	2.1	2.0	-1.9
1957	4.9	5.1	8.8	8.7	0.7
1958	1.3	2.6	7.0	9.6	-4.8
1959	4.5	6.8	11.3	15.6	-1.6
1960	2.5	3.8	9.2	12.0	-2.5
1961	4.5	5.7	11.6	14.2	-0.7
1962	6.6	6.6	11.5	11.8	3.3
1963	1.0	0.7	4.2	3.6	-1.1
1964	4.2	4.3	7.2	7.6	2.3
1965	3.7	3.8	6.4	6.5	2.3
1966	4.5	3.8	7.0	4.7	3.1
1967	6.6	7.1	12.9	14.8	3.3
CUMULATIVE AVERAGE ANNUAL PERCENTAGE CHANGES					
1952-1967	3.2	3.7	7.3	8.1	-0.2
1953-1967	3.2	3.6	7.3	8.1	-0.2
1954-1967	3.4	3.8	7.7	8.5	0.0
1955-1967	3.7	4.2	8.2	9.2	0.2
1956-1967	4.0	4.6	8.8	9.8	0.4
1957-1967	3.9	4.5	8.8	10.0	0.3
1958-1967	4.2	4.7	9.0	10.0	0.9
1959-1967	4.2	4.5	8.7	9.3	1.2
1960-1967	4.4	4.6	8.6	8.9	1.8

Source: R. E. Olley, "Productivity Gains", p. 31.

indexes.

A more rigorous comparison can be done with parametric statistics and with the use of an F test. A summary of the descriptive statistics for each of the index series and for the results of the F test are shown in Table XII. From these statistics it can be seen that productivity indexes have been very similar and from the results of the F test the hypothesis that the variance of the two populations is equal can be accepted at the .01 level of significance. These results indicate that the results are drawn from approximately the same population and the conclusion is that AGT is generally representative of the industry.

A comparison of the partial indexes shows that Bell Canada labour productivity grew at a faster rate than AGT's and that AGT's capital productivity grew at a faster rate than Bell Canada's. The result is that total productivity has grown at about the same rate. These kind of results are not unpredictable if one notes that gross capital investment, the vehicle for technical change, grew by 450 per cent at Bell Canada and by 600 per cent at AGT for the same time period and by 800 per cent at AGT for the full 20 year period. The difference of AGT's capital productivity results may then be explained as a result of the more rapid assimilation of technological changes that are both capital saving and labour saving.

Limitations

The data employed for productivity indexes is consistent with that employed by Olley in his study but there are some limitations

TABLE XII

DESCRIPTIVE STATISTICS

	Index	Mean	Range	Min.	Max.	St.Dev.	Var.	Sd.Error
Capital								
	AGT	91.8	17.7	82.3	100.0	5.4	29.2	1.4
	Bell	94.9	15.0	87.8	102.8	4.9	24.4	1.2
Labour								
Weighted	AGT	67.7	59.5	40.5	100.0	20.4	417.2	5.1
	Bell	56.8	68.7	31.3	100.0	22.5	506.8	5.6
Unweighted	AGT	75.1	49.2	51.1	100.3	17.4	304.0	4.4
	Bell	58.7	65.2	34.8	100.0	20.9	437.1	5.2
Total								
Weighted	AGT	78.5	41.2	58.9	100.0	13.9	193.3	3.5
	Bell	74.3	42.0	58.0	100.0	13.5	182.3	3.4
Unweighted	AGT	83.4	31.9	68.1	100.0	11.1	123.3	2.8
	Bell	76.0	38.1	61.9	100.0	11.9	141.9	3.0

F TEST

H_0: $S_1^2 = S_2^2$

H_1: $S_1^2 = S_2^2$

Region of rejection at .01 level of signifigance =(low) 0.284 to (high) 3.52

	S/S	Result
Capital	0.83721	Accept H_0
Labour-Unweighted	1.21477	Accept H_0
Labour-Weighted	1.43793	Accept H_0
Total-Unweighted	0.94345	Accept H_0
Total-Weighted	1.15061	Accept H0

with this method that should be recognized prior to use of these indexes.

First, for labour inputs, valuation in terms of 1967 wage rates prior to the base year appear to overestimate the wage bill and underestimate it in the period subsequent to the base year. Such a valuation procedure adjusts for price level changes but does not adjust for the effect of changes in real wages that result from such things as productivity gains.

The estimate of the rate of return on the capital stock seems somewhat arbitrary, the criteria being that it distributes income between the two input factors during the base year. In this industry, since rate of return is regulated, they correlate to price level changes and the use of a constant rate appears to overstate the rate of return in earlier years. These problems could be largely ameliorated if net investment had been used as a basis for measurement.

Finally, both of the above problems are related to the problem of choosing a base period and a weighting system. Changes in quantities and in prices tend to be negatively correlated i.e. the purchases shift to items that are relatively cheaper. Thus the more recent the base period, the smaller the apparent increase in real aggregates. While this effect should be minimal because outputs and inputs are similarly affected the point is that weighting conventions have some affect on aggregate movements.

While there are some limitations to the method of preparing the data no change has been made because of the underlying consideration that they should be comparable to those used in the Olley study. At

a later point, for the purpose of estimating the production function, some changes may be required.

Conclusions

Gains in labour productivity over most of the period have been impressive but not spectacular. Only in the final two years of the study, when the absolute number of permanent employees dropped by over 150 with output continuing to expand, has there been a spectacular increase in labour productivity. Capital productivity has also shown a continuing rise throughout the period. What is most surprising is that capital productivity continued to grow even though as a proportion of total inputs it changed from approximately 40 per cent in 1950 to almost 65 per cent in 1969. These results indicate that considerable technological advances were made in the capital inputs throughout the period. Total factor productivity reflects the changes of both the partial indexes. While both partial indexes exhibit a tendency to diverge from results of a comparable Bell study, the total factor productivity index appears to cover an almost identical range. The comparison of results indicates that both sets of indexes are from the same general population. At this point a tentative conclusion is that the industry productivity changes apply to both but that the individual firms tend to employ a different technology.

Over extended periods of time and even during intermediate time periods, such as the one under study, productivity rates reflect changes in the technology and organization of production. In this study one can already see some profound changes that have taken place.

Capital has changed from the secondary to the primary factor of production. Proportionately increasing capital intensity of the production process along with technological improvements has caused a great increase in the productivity of labour. Essentially the production process has changed from a labour intensive to a capital intensive one. Productivity gains will be analyzed in general in the next chapter. They may be a result of a reduction in unit costs which can be achieved internally, for example with organizational changes, or from external sources such as a higher quality labour supply or more advanced technology available to the industry.

CHAPTER V

FITTING THE PRODUCTION FUNCTION

Introduction

Productivity indexes, as pointed out earlier, serve a useful purpose but are limited in their application. On the other hand, if a production function can be successfully fitted with the data provided by the firm then, depending on the restrictive assumptions made, estimates of the economic parameters can be made and a powerful tool for prediction can be developed. This study specifies, a priori, the model to be used. This chapter also tests whether such a model could be used to fit data provided by AGT to the production function and therefore whether or not it is a useful tool for corporate use.

In this chapter the first section will present the data to be used in the regression analysis. The second section will discuss the estimation of the 'side relation' discussed in Chapter III and the third section will be a discussion of the estimates of the CES function. The fourth section will discuss the results of the model and the final section will present conclusions.

Data

Essentially the same data as that used for the productivity relationships will be employed. However, different estimates or variations of the capital and labour inputs will be used to determine

what effect, if any, these variations have in estimating the side relation.

Two measures of capital are employed. The method of measuring gross capital stock is not changed from the previous Chapter. While authors such as Solow[1] and Ferguson[2] recommend that capital stock should be adjusted for capacity utilization by means of a 'utilization' factor, this study is at the level of the firm and because of the nature of its business it would seem better not to adjust capital stock. Net capital stock is derived from gross capital stock by subtracting the value of depreciation from the gross capital stock. Depreciation for this purpose is derived by adjusting reported depreciation by means of a composite of the previously employed plant price indexes. The constant dollar estimates of gross capital stock are reported in column 2, depreciation in column 3 and net capital stock in column 4 of Table XIII.

Two measures of labour are used: total manhours worked and total manhours worked with a simple adjustment to reflect changes in the quality of the labour force. These are repeated from Table V and are shown in columns 5 and 6 of Table XIII.

Gross value added is repeated from Table VII and net value added is derived by subtracting depreciation estimates from gross value added. These are shown in columns 7 and 8 of Table XIII.

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[1] R. M. Solow, "Technical Progress, Capital Formation and Economic Growth", The Journal of Economics, 52 (1961) p. 77.

[2] C. E. Ferguson, "Time Series Production Functions and Technological Progress in American Manufacturing Industry", The Journal of Political Economy, 73 (April 1965) p. 138.

A proxy for the rental value of capital was obtained by using the interest rates paid by AGT on their long term debentures. No attempt was made to measure actual yields on bonds because the spread was usually constant and its use would have no affect on the final results. The wage rates were obtained directly from AGT accounting data and represent the average hourly wage of the year. Wage rates are shown in column 9 and interest rates are shown in column 10 of Table XIII. The ratio of interest rates to wage rates is calculated and shown in column 11 of Table XIII.

Expansion Path Estimation

The most common form used to estimate the CES function has been to employ the expansion path function or as it has been referred to earlier the 'side relation'. It is repeated here in its logarithmic form:

$$\log N/C = \sigma(1-\lambda) \log \kappa' + \sigma \log q/w + \lambda \log (N/C)_{-1}.$$

The estimates $\hat{\sigma}$ and $\hat{\lambda}$ are obtained by means of least squares. The multiple regression computer program employed was an SPSS package.[3]

This expansion path relation is derived from the marginal rate of substitution introduced previously which includes σ as a parameter. It can be restated as follows;

$$\mu = \kappa'^{-\sigma} \rho^{\sigma}$$

where $\rho = q/w$

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[3] N. H. Nie, D. H. Bent and C. H. Hull, Statistical Package for the Social Sciences. (New York: McGraw Hill, 1970) p. 174. The multiple regression method can employ either the normal multiple regression procedure or step wise regression. Both have an additive random error term and both methods were employed in the estimation procedures. In either case the underlying procedure is Gauss elimination with some row and column interchange.

TABLE XIII

PRODUCTION FUNCTION DATA

Year	Measures of Capital			Measures of Labour		Value Added		Factor Prices		Ratio of Factor Prices
	Gross Capital Stock	Deprec- -iation	Net Capital Stock	Manhours Worked	Manhours Quality Weighted	Gross Value Added	Net Value Added	Average Wage	Interest Rates	
Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8	Col. 9	Col. 10	Col. 11
1950	57,343	1,166	56,177	2,762	1.992	8,588	7,826	$1.22	0.03%	0.0246
1951	63,631	1,125	62,505	3,247	2,376	9,841	8,716	1.26	0.03	0.0238
1952	71,465	1,140	70,324	3,815	2,832	11,349	10,209	1.38	0.03	0.0219
1953	80,442	1,421	79,020	4,141	3,100	13,160	11,739	1.42	0.035	0.0246
1954	88,475	1,942	86,533	4,239	3,262	14,879	12,937	1.53	0.035	0.0228
1955	103,843	2,613	101,229	4,342	3,412	17,032	14,419	1.65	0.035	0.0212
1956	117,797	2,978	114,819	4,870	3,904	19,957	16,979	1.61	0.04	0.0248
1957	144,086	3,760	140,325	6,031	4,727	22,025	18,265	1.64	0.04	0.0244
1958	164,350	4,131	160,218	6,442	5,186	24,301	20,170	1.75	0.05	0.0285
1959	179,302	4,803	174,498	6,600	5,445	27,732	22,929	1.96	0.055	0.0281
1960	200,656	6,254	194,401	6,239	5,243	30,980	24,726	2.13	0.055	0.0258
1961	220,264	7,140	213,124	6,349	5,400	34,805	27,665	2.36	0.047	0.0201
1962	238,022	8,342	229,679	6,683	5,778	40,251	31,909	2.48	0.055	0.0222
1963	260,043	9,583	250,459	7,387	6,541	44,793	35,210	2.53	0.053	0.0213
1964	287,449	11,444	276,004	7,754	6,893	50,781	39,337	2.71	0.052	0.0194
1965	325,409	13,886	311,543	8,297	7,440	56,119	42,253	2.74	0.048	0.0178
1966	369,794	12,859	356,934	9,137	8,355	65,659	52,800	2.81	0.060	0.0214
1967	411,225	18,560	329,665	10,062	9,426	73,857	55,297	3.12	0.060	0.0192
1968	451,036	22,357	428,660	9,807	9,445	84,848	62,491	3.38	0.062	0.0185
1969	487,195	24,926	462,268	9,393	8,950	93,492	68,566	4.01	0.073	0.0184

Notes: 1. All figures are in thousands (000) except columns 9, 10 and 11. Measures of capital and value added are in constant dollars.

The expansion path states explicitly that decisions on a firm's relative factor inputs are a function of the relative factor prices. However, as Brown states[4] the desired labour-capital ratios cannot be obtained instantaneously for any variation in factor prices and limitations placed on the equilibrium condition by considerations of number of observations under study leads to the entrance of additional variables into the regression equation. These additional variables explain the difference between long and short run elasticities of substitution.

Additional constraints are imposed by the rigidity of the capital in use and by the technology of the moment. Both act as a constraint on the long run elasticity of substitution. This is further explained in the following quote from Brown:

> ...if the existing technology permits capital to be substituted with relative ease, but the existing stock of capital changes very little, if at all, to market stimuli, then the difference between the two elasticities is large. There is little we can say about the size of the long-run elasticity - it is what the technology determines it to be. But the reasons for the rigidity of the capital stock can be made manifest. They include the average economic life of the capital items, the rate of investment, and the range of variation of the application of labour to the existing capital stock. As an approximation, one can say that changes in depreciation and the rate of investment cannot effect substantial variations in the capital-labour ratio in the short run, say, one or two years. We are then left with a technological reason for the rigidity of the capital stock. Hence, the major part of the difference between the two elasticities is attributable to the characteristics of the technology.[5]

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[4] Brown, Theory and Measurement, p. 65.

[5] Brown, Theory and Measurement, p. 67.

The additional constraints imposed by the short run can be entered into the regression model explicitly. If changes in factor ratios are not responsive to changes in factor prices in the short run it is difficult to substitute labour for capital in the current period in response to changes in relative factor prices. The more influential past factor price ratios are on present factor input ratios, the more resistant to change will be the installed capital labour ratio to changes in current factor price ratios. This is entered as a distributed lag into the regression and the regression estimate of λ represents the degree of resistance or the rigidity parameter. This parameter is technologically determined by the willingness of installed capital to be substituted for labour for any given output. (The more automatic or durable the equipment the more resistant it is to substitution.) This produces the short run expansion path function.

If the trend term mentioned previously is included in the regression then the regression coefficient of t, the parameter estimate of α, represents the rate at which the capital intensity parameter κ' changes. A positive sign for the regression coefficient indicates a capital using technology and a negative sign indicates a capital saving technology.

The SPSS multiple linear regression package was used with various specifications of the inputs to derive estimates of α and κ. It was found that whether one, two or all three variables were entered into the equation no satisfactory estimates could be obtained. The results of these estimating attempts, including changes

in the variables and data entered into the estimating equation are shown in Table XIV.

Attempts at fitting the data by means of multiple regression can be explained by analyzing the effects which each of the independent variables had on the estimating equation. First, the estimating equation was highly restricted when it was assumed that λ was equal to zero (0) and a trend term was not included. The resulting estimate of σ, obtained by regressing q/w on N/C, did not prove satisfactory. The input factor ratio did not prove to be responsive to changes in factor prices. The R^2 value for q/w was approximately 0.35 and hence offerred little explanation for variation in the dependent variable. The results are labelled as option 9 in Table XIV.

Effects of variations in the measurement of the factor-price ratio can also be seen from options 7,8 and 12 of Table XIV. The use of constant, Ferguson type,[6] rates of return as oppossed to the proxy interest rates has an adverse effect on the results of estimating the side relation. The substitution of the Ferguson constant rate of return had the effect of reducing the explanatory effect of q/w or the correlation between N/C and q/w. Consequently, when constant rates of return were employed in the factor price ratio the F-statistic became insignificant and q/w would not enter the regression. On the other hand the ratio q/w, derived from using proxy interest rates and the average wage rate, remains relatively constant throughout the 20 year period and, as mentioned previously, it also had little ,

[6] Ferguson "Time Series Production Function", p. 138.

but better, explanatory effect.

Inclusion of the variable N/C_{-1} in the regression was made in order to explain the short run rigidity of factor ratios with respect to factor prices. When the regression equation used the two independent variables N/C_{-1} and q/w the past factor ratio N/C_{-1} explained approximately 98 per cent of the variation in the dependent variable. The change in R^2 produced by the addition of q/w was very small.

As well as adding N/C_{-1} as an independent variable, different measures of capital used as part of this ratio were experimented with. It was found that a change in the measure for capital stock did not significantly affect the results. Options 1,2 and 4 shown in Table XIV used gross capital stock, net capital stock and the Ferguson measure for gross capital services[7] respectively. The results reported in the table show no significant difference between the results obtained using the various measures of capital stock. This can be shown in the context of the regression. First N/C_{-1} explains approximately 98 per cent of the variation in N/C. Changing the measure of capital within each of these ratios does not change the higher degree of correlation between them. In other words, all three measures represented a similar degree of capital rigidity which was a better explanation for the variation in N/C than were changes in factor prices which remained relatively constant.

Finally if time was entered into the regression as an independent

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[7] Ferguson, "Time Series Production Functions", p. 138.

TABLE XIV

SIDE RELATION COEFFICIENT ESTIMATES OBTAINED FROM LINEAR REGRESSION

Option	Variables				Inclusion	# of		Normal Coefficients			Simple R			F Stat. for Last
	N	C	Q	Time	Order	Cases	Constant	σ	λ	α	σ	λ	α	Variable Entered
1	$	Gross	Proxy	no	Step	20	-0.0112	-0.0156	0.9929		0.6471	0.9825		(q/w) 0.063
2	$	Net	Proxy	no	Step	20	0.0082	-0.0186	0.9943		0.6439	0.9819		(q/w) 0.037
3	$	Net	Proxy	yes	Step	20	26.8550	0.0063	0.3934	-0.5995	0.6439	0.9819	-0.9862	(q/w) 0.018
4	$	G.Ser	Proxy	no	Step	20	-0.0424	-0.0136	0.9915		0.6462	0.9824		(q/w) 0.048
5	Hours	Gross	Proxy	no	Step	20	0.0475		0.9830			0.9839		q/w was insig.
6	Hours	Gross	Proxy	yes	Step	20	27.3929	0.0251	0.3781	-0.6009	0.6553	0.9830	-0.9867	(q/w) 0.268
7	Adj.	Gross	Con.	yes	Step	20	24.2338		0.2484	-0.7359		0.9975	-0.9919	q/w was insig.
8	Adj.	Gross	Proxy	yes	Step	20	24.1439	-0.0432	0.2897	-0.7353	0.6023	0.9175	-0.9719	(time) 16.421
9	Adj.	Gross	Proxy	no	Simple	20	0.0799	0.6023			0.6023			(q/w) 9.679
10	Adj.	Gross	Proxy	yes	Normal	20	32.2090	-0.0130		-0.9801	0.6023		-0.9719	(q/w) 0.030
11	Adj.	Gross	Proxy	yes	Step	20	32.2090	-0.0130		-0.9801	0.6023		-0.9719	(q/w) 0.030
12	Adj.	Gross	Proxy	yes	Normal	20	24.1439	-0.0432	0.2779	-0.7353	0.6023	0.9475	-0.9719	(N/C) 2.170
13	Adj.	Gross	Proxy	no	Step	10	0.0235		0.8869			0.8869		q/w was insig.
14	Adj.	Gross	Proxy	no	Step	10	-0.1204		0.9246			0.9246		q/w was insig.

NOTES:
1. Average wage rates were used in the factor price ratios for all options.
2. The $ sign signifies unadjusted labour in terms of 1967 dollars.
3. Hours signifies unadjusted manhours.
4. Adj. signifies manhours adjusted for quality.
5. Capital series used is signified by Gross for gross capital stock, by Net for net capital stock and by G.Ser for gross capital services.
6. Time is shown as having been included or not included.
7. The type of regression is signified by step for stepwise regression, by normal for normal regression and by simple for simple regression.

variable it explained approximately 98 per cent of the variation in the dependent variable and the addition of q/w and N/C_{-1} produced only a small change in R^2. This result was caused because the time variable shows the greatest variation, N/C_{-1} shows the second highest degree of variation and q/w remains relatively constant over the 20 observations.

In all cases the normalized regression coefficients for time had a negative sign which, according to Brown's model, suggests that technological progress, as measured by the capital intensity parameter κ', is capital saving. This seems to contradict observed increases in capital intensity but can be explained by the following reasons. First, while capital intensity is increasing, the technology embodied in capital can still be capital saving. This will be the case if the rate of growth in output exceeds the rate of growth in intensity. In such a case capital is being substituted for labour but capital itself is also becoming much more efficient. When the second effect is greater than the first both increasing capital intensity and capital saving technological change will be observed. A new automatic machine will replace labour but it may also be capable of handling a much greater volume of output than the older manual machine. This capital saving bias may also have been increased through changes in organization and marketing that have increased the utilization rate of existing capital. Again the efficiency of capital is being increased.

In general all of the results obtained from this side relation proved to be either inconclusive or unsatisfactory. As a result, continuation with this method of estimation was halted and an alternative method, discussed in the following section, was employed.

Because no satisfactory estimates of α and κ could be obtained the model could not be used to estimate the second two parameters of the CES function, the efficiency parameter, γ, and the returns to scale paramenter, ν.

Also, because of the failures of this method this model was not extended into the finite differencing method for discerning technological changes as no significant estimates of these parameters could be achieved with less than 20 observations. Further comments about this problem will be made in a following section of this chapter.

All attempts to employ this method proved unsatisfactory eventhough the data was, conceptually, proper. The unsatisfactory results obtained are similar to those found by Bodkin and Klein at a higher level of aggregation.[8] Bodkin and Klein results are reported in Appendix III. For Bodkin and Klein a straight (linear) regression, either with an additive or a multiplicative error, produced results that were unsatisfactory and were rejected on the basis of single equation bias instead of economic theory. The unsatisfactory results obtained suggests that the model cannot accomodate non-linearity in the data.

Direct Estimation of the CES

Because of the poor results obtained from the side relation for the CES function, the alternative non-linear maximum likelihood

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[8] R. Bodkin, and L. Klein, "Non-Linear Estimation of Aggregate Production Functions", Review of Economics and Statistics, 49 (1967) p. 37.

method reported by Brown was employed. This method employs a direct non-linear least squares method of estimation and was reported in detail in Chapter III.

The BMD-X85 program[9] was used to computerize this model and sample inputs and outputs produced using various combinations of inputs are included in Appendices III and IV respectively. The results of the various regressions using this method are summarized in Table XV. A fit was obtained for a 20 year period, 17 year period, and 15 year period.

In using this approach, the original estimates were obtained partly from a study done by Bodkin and Klein and partly from estimates obtained earlier in this study in attempting to estimate the side relation. They are given in row one of Table XV. The major problem in pursuing this approach was that convergence was only obtained by confining the technological efficiency parameter, γ, to a figure less than 1.0. Nevertheless as can be seen from the estimated values appearing in Appendix IV a 'reasonable' estimate of the dependent variable can be obtained. The changes in the individual parameters can be examined independently.

The technological efficiency parameter, γ, was artificially constrained in order to achieve convergence and therefore remains constant throughout for all variations. Essentially this parameter implied the rate of disembodied neutral technological progress and is held constant at a rate of .9 per cent per annum. This compares

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[9] W. J. Dixon(ed.), Biomedical Computer Programs-X Series Supplement,(los Angeles: University of California Press, 1969). This program employs stepwise Gauss-Newton iterations to obtain least squares fit.

TABLE XV

CES PARAMETER ESTIMATES OBTAINED FROM NON-LINEAR REGRESSION

Options	Number of Obser- vations	Parameters						Error Mean Square	Comments and Missing Observations
		γ	κ	α	r	ν	σ		
Orig.		30.00	0.45500	0.450	3.020	1.35900	0.68965		
1	20	0.0090	0.22492	0.49787	3.0171	1.50191	0.66760	1,780,100	Non-Convergence
2	17	0.0090	0.22671	0.49664	3.0171	1.49841	0.66816	860,190	1967,68 and 69
3	16	0.0090	0.19858	0.49926	3.0151	1.50454	0.66699	948,640	1966,67,68 and 69
4	15	0.0090	0.19930	0.49972	3.0150	1.50666	0.66679	775,570	1959,66,67,68 and 69

Notes: The estimating equation is; $X = \gamma[\kappa C^{-\alpha} + (1-\kappa)N^{-\alpha}]^{-\nu/\alpha}$

where the variables are; Labour = N = manhours worked per year adjusted for quality.
Capital = C = Gross capital stock in constant dollars.
Value added = V = Value added per year in constant dollars.
$r = \nu/\sigma$

with rates reported by Bodkin and Klein of 1.49 per cent and 1.37 per cent.[10]

Returns to scale are represented by the parameter, ν, and are estimated to be approximately 1.50 for all variations. This strongly suggests increasing returns to scale because it is significantly above the constant returns to scale value of unity. This result compares favourably to the non-linear results of Bodkin and Klein of 1.24 and 1.36.[11]

Estimates of κ show a significant increase as the number of observations change from 16 to 17. As σ remained constant and κ increased and also as the rate of growth of capital increased more rapidly than that of labour, output had to increase rapidly. This would appear to have been the case at AGT. The estimates of κ cover a range from 0.199 to 0.266 and are substantially lower than the estimates for the American economy reported by Bodkin and Klein of 0.631 and 0.447.[12] These low estimates represent an average for the 20 year period and consequently they indicate a labour intensive situation.[13] The fact that κ is increasing agrees with the changes taking place towards a capital intensive process.

Finally, for all variants of the function the elasticity of substitution, σ, is approximately 0.66. This result is significantly different from unity and compares with the non-linear estimates of

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[11] Bodkin and Klein, "Non-Linear Estimation," p. 39.

[12] Bodkin and Klein, "Non-Linear Estimation", p. 39.

[13] Brown, Theory and Measurement, p. 48.

Bodkin and Klein of 0.47 and 0.68.[14] This also compares with the results of Arrow, Chenery, Minhas and Solow which are reported at their corrected value of 0.61.[15] This constant elasticity of substitution indicates that the major factor affecting changes in κ comes as a result of bias in technological progress.

Measurement of Technological Change

Brown's method of measuring technological change calls for fitting the production function to contiguous time periods and then comparing the estimates by means of a finite differencing approach. As Brown points out[16] the difficulty lies in isolating epochs which characterize a different technology. In the case of this survey the task has proven impossible for the following reasons.

First, no statistically significant results were obtained by the linear side relation method of estimating the function with anything less than the full 20 year period. Second, convergence with the non-linear estimation techniques was obtained only by holding the efficiency parameter constant and this remains true for any portion of the 20 year period. In doing this it is assumed that there would be no non-neutral technological over the 20 year period. This assumption would bias further attempts at measuring technological change by the finite differencing approach. Therefore, continuing with Brown's

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[14]Bodkin and Klein, "Non-Linear Estimation", p. 39.

[15]K. Arrow, B. Minhas, H. Chenery, and R. Solow, "Capital Substitution and Economic Efficiency", Review of Economics and Statistics. 43 (Aug. 1961) p. 225.

[17]M. Brown and J. Popkin,"A measure of Technological Change and Returns to Scale", Review of Economics and Statistics. 44(1962) p. 405.

model proved impossible because no satisfactory estimates for short time periods could be made.

Conclusions

The original data prepared for estimating productivity indexes is usable for estimating production functions. Attempts to employ the expansion path method for estimating the CES production function proved inconclusive and this method could not be extended to the measurement of technological 'epochs'.

A reasonably close fit of the CES production function was obtained using non-linear least squares technique. Unfortunately, the only means of obtaining convergence was to hold the efficiency parameter at a level less than 1.0. A fit of the production function employing this method, for different time periods, found a general trend in the parameters estimated.

Finally, the estimated parameters indicate that neutral technological change played a subsidiary role. Most of the improvements in the rate of growth of output seems to have come about either as a result of increasing returns to scale or by increasing the rate of growth of capital during a period when the capital intensity parameter has increased and the elasticity of substitution remained stable.

CHAPTER VI

EVALUATION AND CONCLUSIONS

General

The original model required the preparation of data which could be used both for the preparation of productivity indexes and the fitting of the CES production function. The fitting of the CES function was done in order to derive the role played by technological change in the communications industry. This final chapter will summarize the limitations and shortcomings of the model and the results of the study.

Summary of Results

The results of the study have been reported in Chapter IV and V. The preparation of productivity indexes was straightforward and followed the methods as outlined by Olley. The results of the productivity indexes, which differed in some ways from those reported for 'Bell Canada', for the most part, were substantivley similar. In general they show that productivity at AGT has been growing at a faster rate than the economy as a whole.

The results from fitting data to the CES production function, while not as clear cut, are also substantial. Estimates of the CES parameters using the 'side relation' or indirect method did not prove successful. The results, very low or negative estimates for σ and very high estimates of λ, the rigidity parameter, were not realistic and another method of estimation was employed. This side

relation method may have been unsuccessful because of non linearities in the data or, as mentioned in Chapter III, because σ is unresponsive to changes in factor proportions.

The non-linear least squares technique has allowed the estimation of all four parameters. These results indicate that increasing returns to scale and increasing capital intensity are the main causes of increasing growth rates. However, these results are tempered by a set of limitations that will be discussed below. Further analysis in terms of 'technological epochs' was precluded because of the inability to satisfactorily measure less than the full 20 year period.

Summary of Limitations

The limitations of the model proposed have all been listed previously and will only be briefly summarized here. There are some major problems with this type of empirical work caused by the aggregation of data into homogenous categories. There are also some problems in specification both for productivity indexes and for production functions. The preparation of indexes implies a production function with constant parameters. The fitting of a function assumes that all factors of production are included and properly specified.

Some of the problems with the results can also be attributed to the weaknesses of the CES function. First because ν combines returns to scale and technological change that alters growth. Technological progress is also represented in the efficiency parameter γ and when this is held constant, as it was in the empirical work

reported here, it may cause an upward bias in ν.

The fact that σ does not change in response to changes in factor proportions is a relevant weakness. The failure of attempts to estimate the CES parameters by an indirect method may be caused because this method makes σ dependent on changes in the ratio of the prices of factor proportions. The empirical results showed this ratio to be constant and the result was an unreliable estimate of σ.

Finally the model does not account for changes in the demand mix so that changes in taste not associated with changes in technology can not be separated from changes due to technological change. The increasing demand had made it impossible to establish utilization rates for the capital stock but the effect of increasing utilization along with higher growth rates can be seen in the negative sign for the coefficient of time when it was included in the side relation regressions.

Conclusions

The overall results of the proposed model are satisfactory. The original intention was to specify a method of measuring productivity, productivity relationships and technological change which could be employed at the level of the firm. Given this objective, moderate success has been achieved.

First productivity indexes have been prepared using a method outlined by Olley but which can be substantitated from other sources. These methods, applied to data gathered from AGT, appear successful. Reasonable measures of labour, capital and total productivity have

been achieved.

Second, fitting data to the CES function to derive estimates of the parameters was also successful. While the model that was specified a priori failed, the alternative technique of non-linear least squares produced a reasonable estimate of the CES parameters.

The inability to measure technological change may be due as much to the time period as to the method employed. If the long-run, or period in which capital can be replaced, exceeds the period of study then no epochs will be found. The attempts to establish epochs within a 20 year period in this industry may have been premature.

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APPENDIX I

DATA GRAPHS

Exhibit II

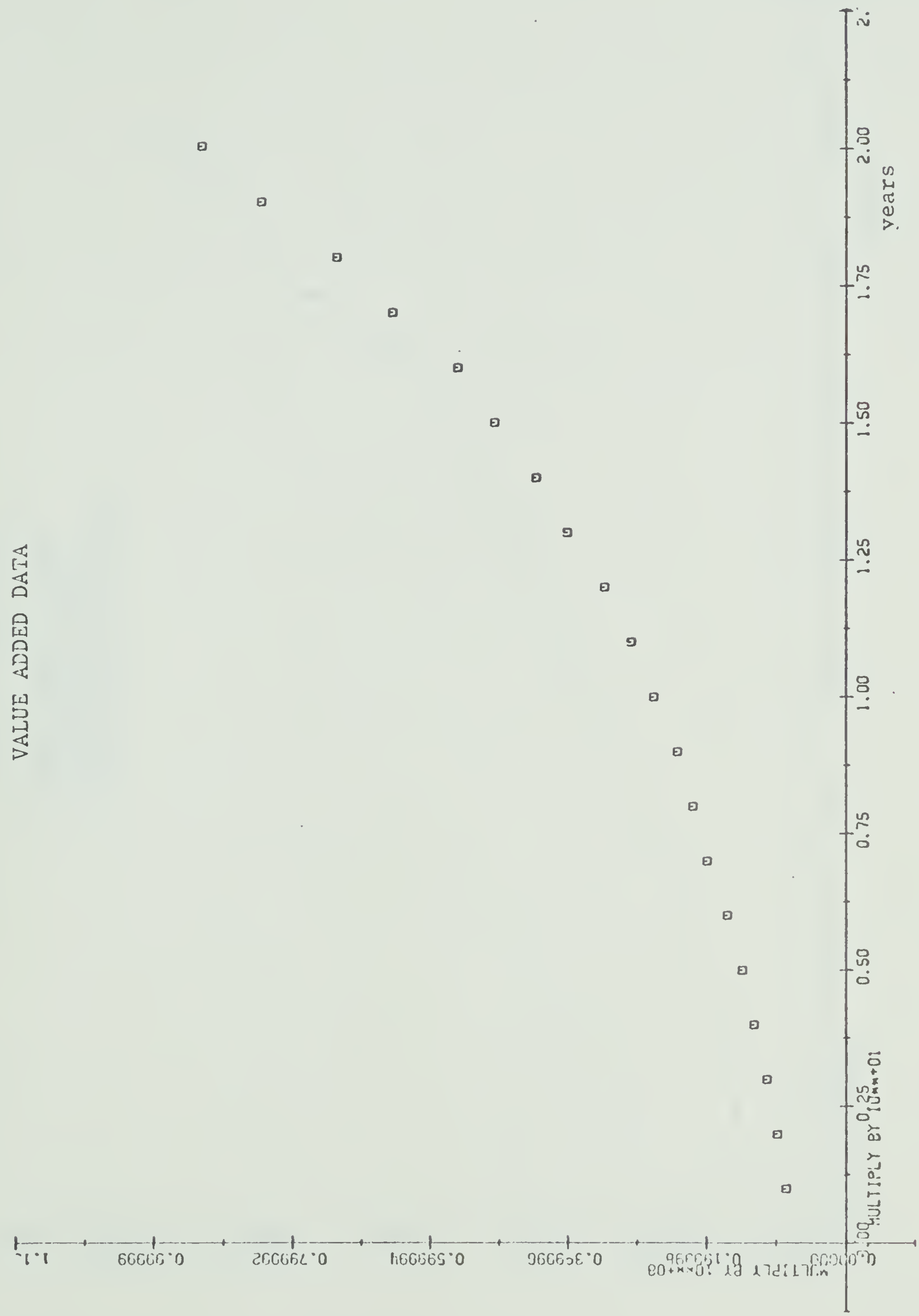

VALUE ADDED DATA
years
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
MULTIPLY BY 10**+01

Exhibit III

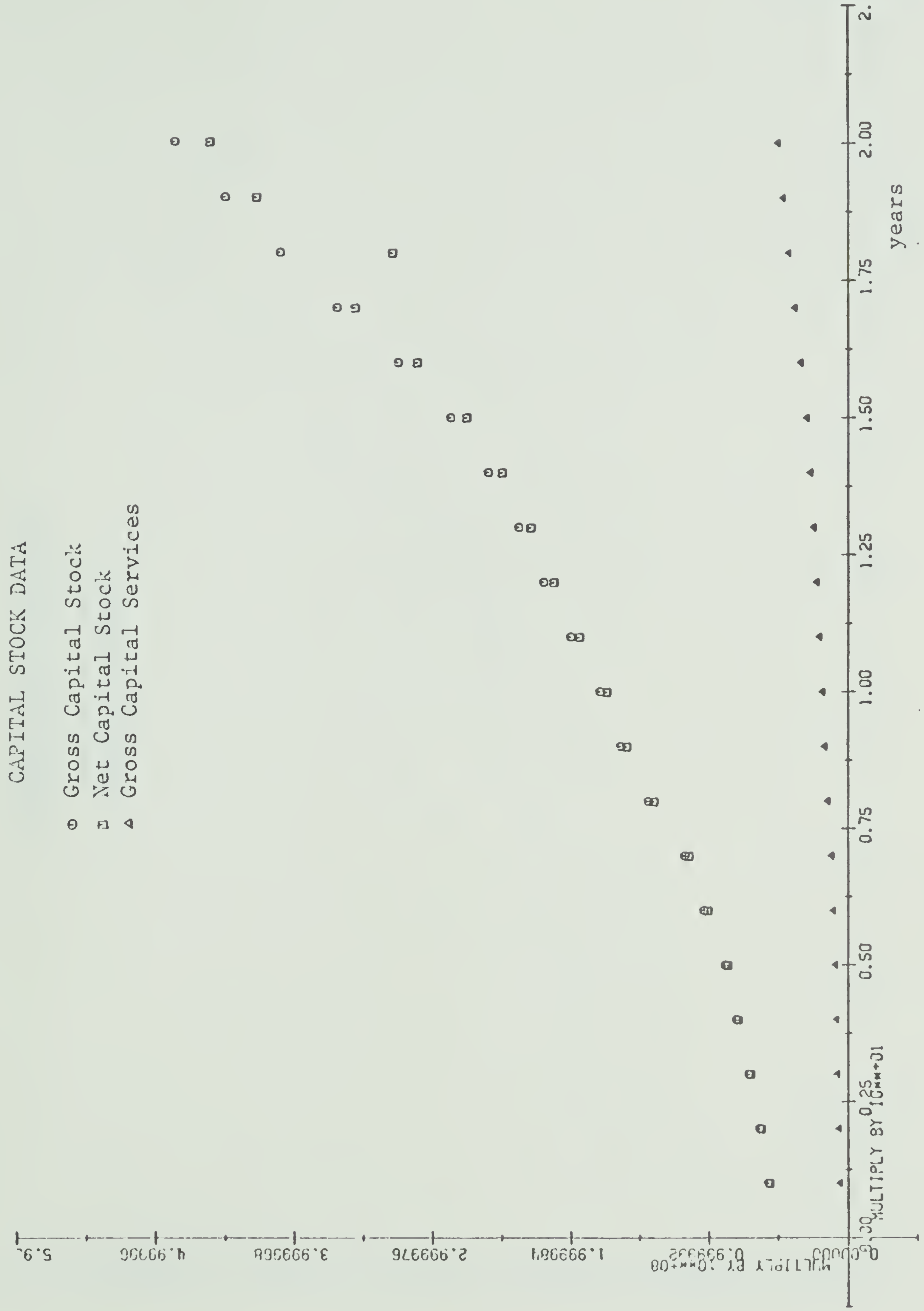

CAPITAL STOCK DATA
Gross Capital Stock
Net Capital Stock
Gross Capital Services
years
MULTIPLY BY 10**+01
MULTIPLY BY 10**+00

Exhibit IV

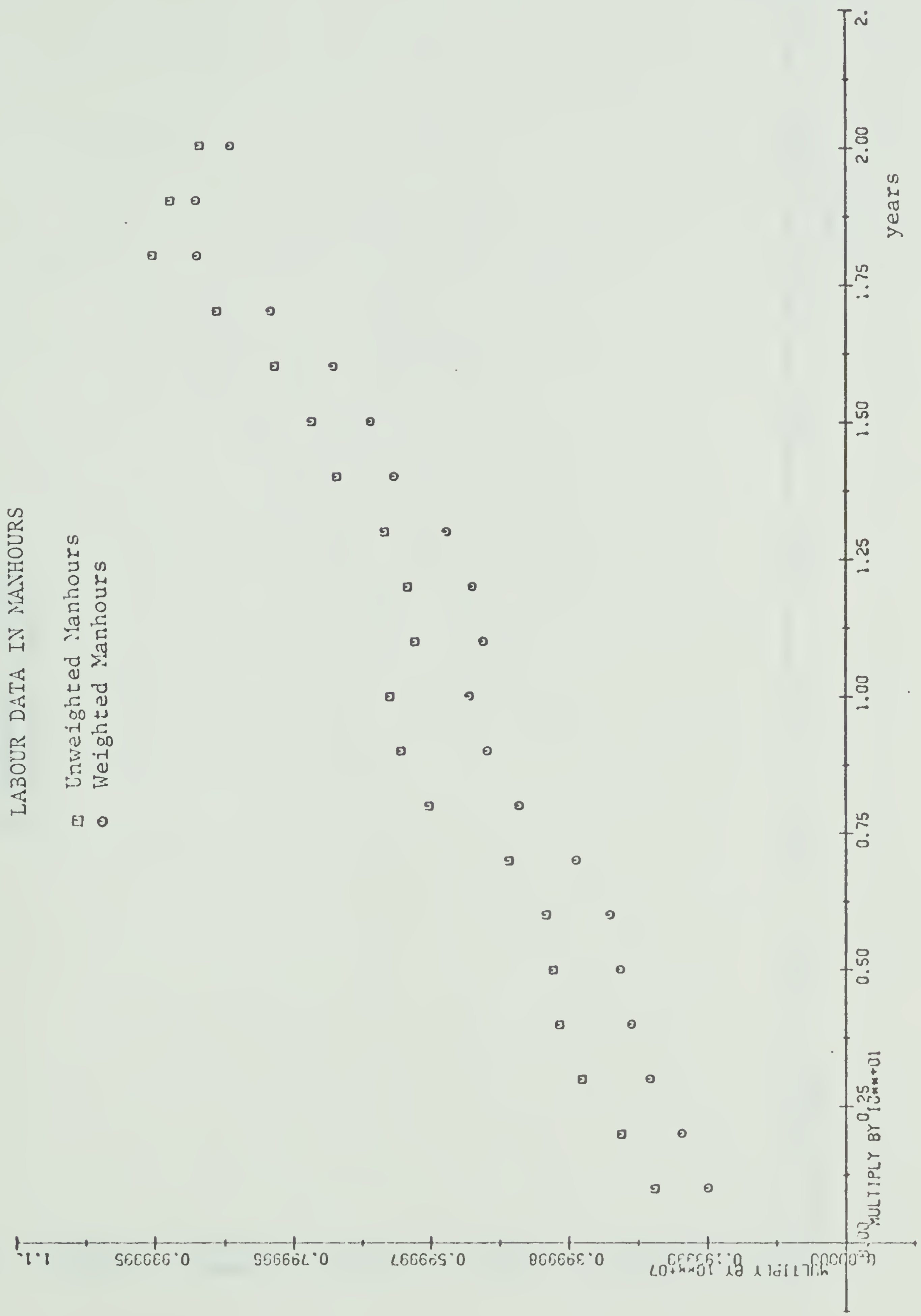

LABOUR DATA IN MANHOURS
Unweighted Manhours
Weighted Manhours
years
MULTIPLY BY 10**+07
MULTIPLY BY 10**+01

Exhibit V

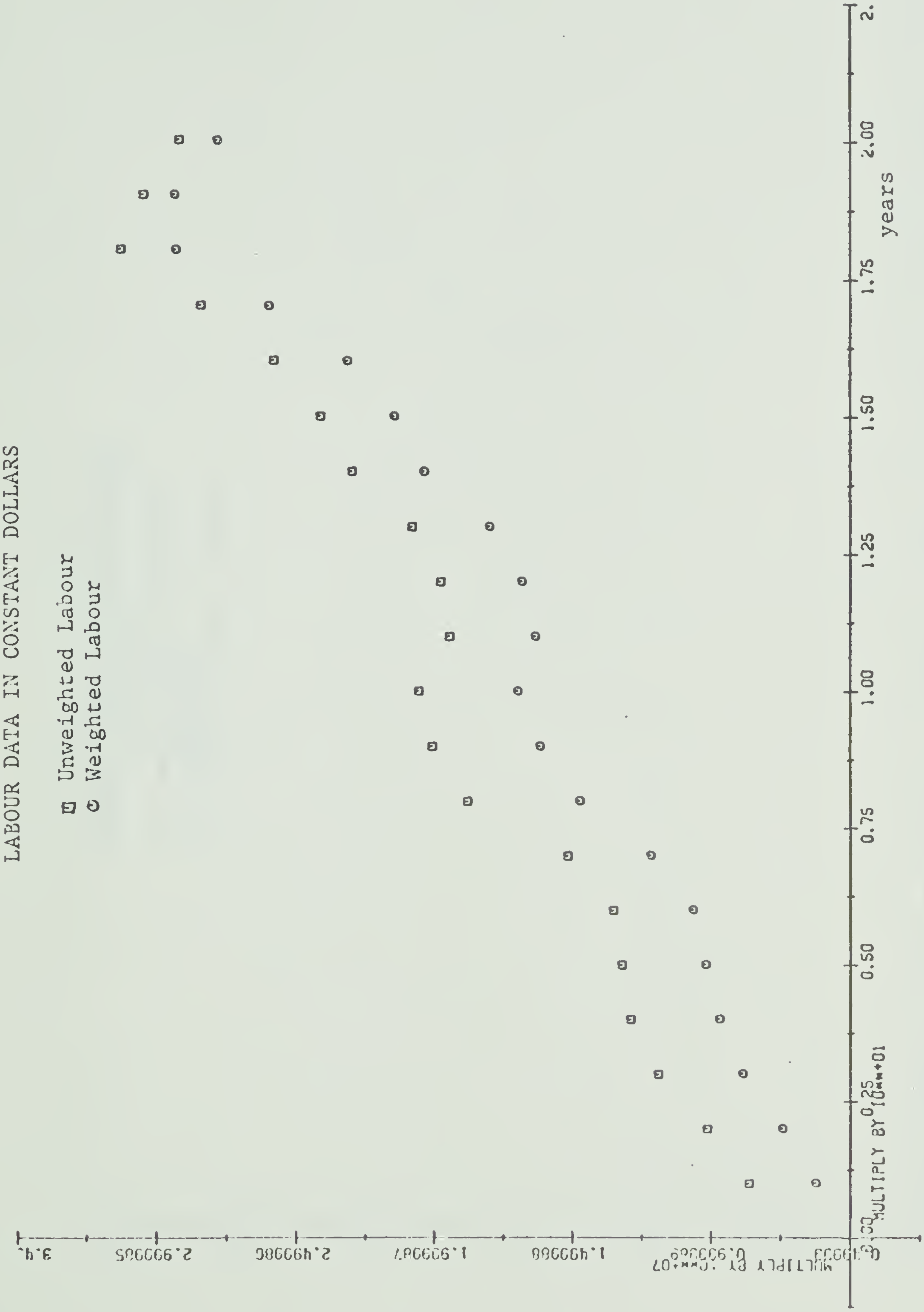

LABOUR DATA IN CONSTANT DOLLARS
Unweighted Labour
Weighted Labour
years
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
MULTIPLY BY 10**+01
MULTIPLY BY 10**+07

Exhibit IV

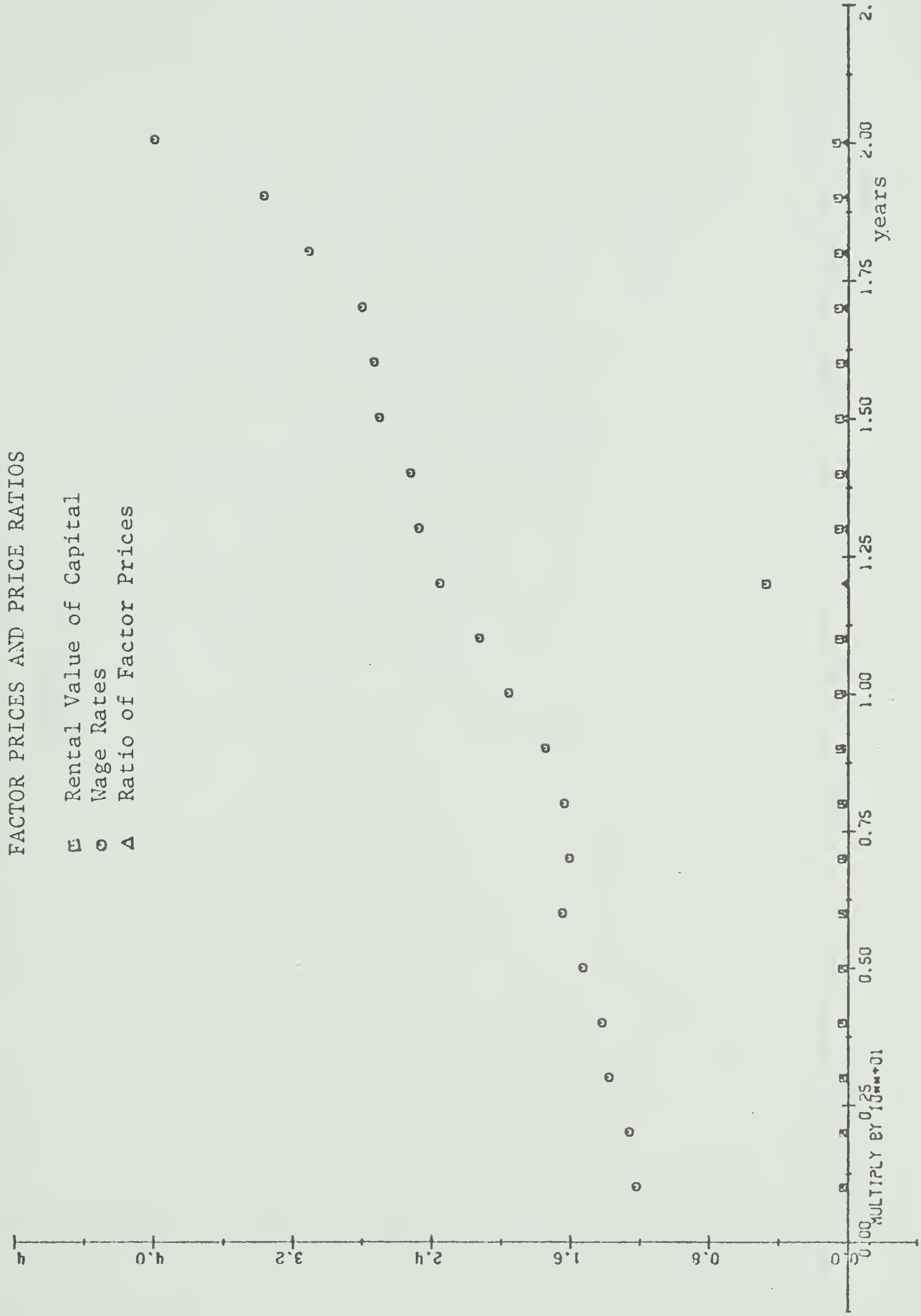
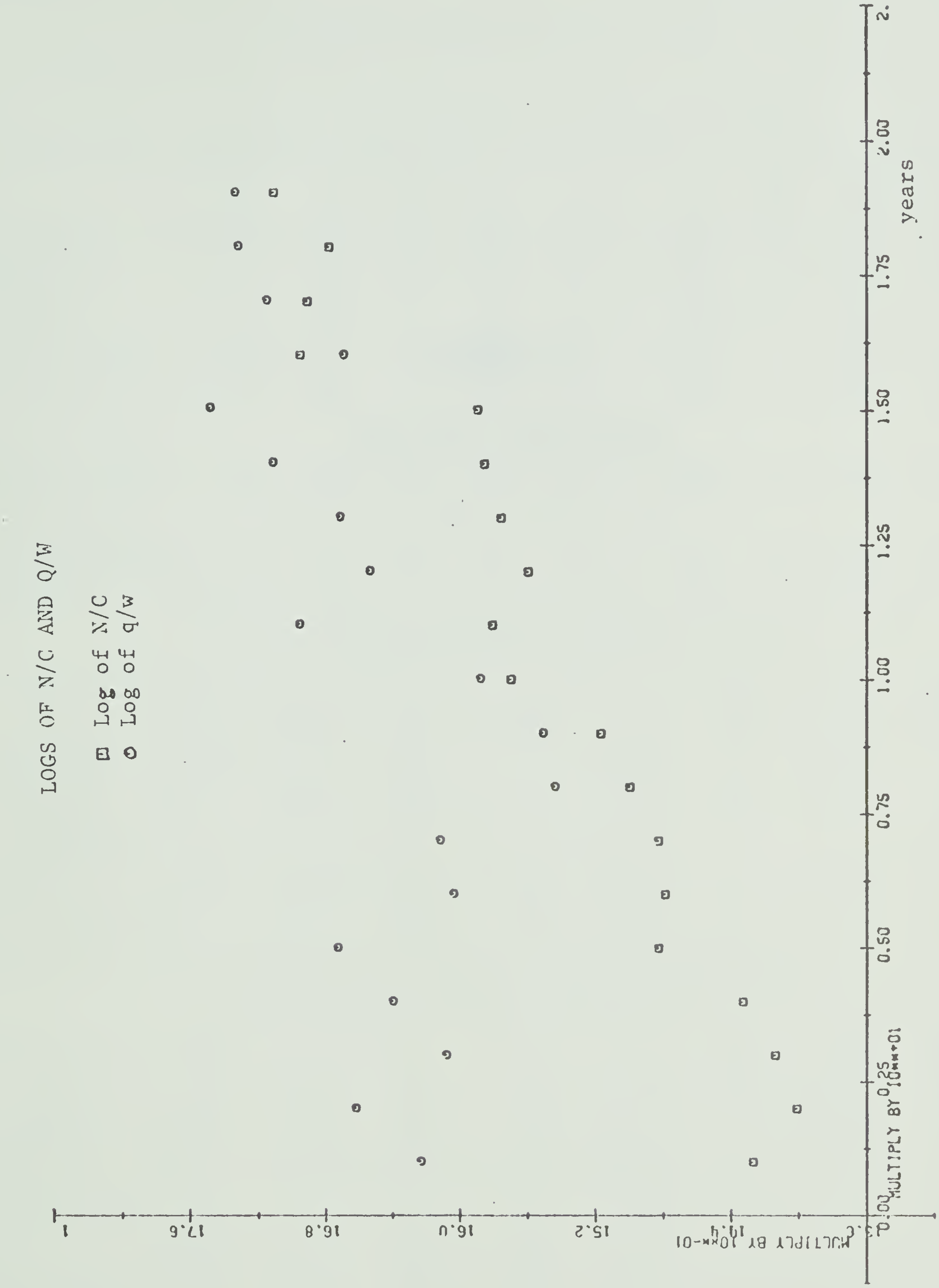

FACTOR PRICES AND PRICE RATIOS
Rental Value of Capital
Wage Rates
Ratio of Factor Prices
years
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.
MULTIPLY BY 10**+01
0
0.8
1.6
2.4
3.2
4.0
4

Exhibit VII


LOGS OF N/C AND Q/W
Log of N/C
Log of q/w
years
MULTIPLY BY 10**-01
MULTIPLY BY 10**+01
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
14.4
15.2
16.0
16.8
17.6

APPENDIX II

BODKIN AND KLEIN RESULTS

Exhibit VIII

RESULTS OF BODKIN AND KLEIN STUDY

Estimates of the Parameters of the CES Production Function

	A or $\log A$ (1)	ρ (2)	$\sigma = \frac{1}{1+\rho}$ (3)	δ (4)	μ (5)	λ (6)	10^{λ} (7)	$\bar{R}^2$ S_u (8)	$\frac{\delta^2}{S^2}$ (9)
Straight regression, multiplicative error	1.6466 (0.1083)	9.593 (7.591)	0.0944	0.9975 (0.0160)	1.210 (0.0156)	0.00675 (0.00036)	1.0157	0.9926 0.01346	1.395
Straight regression additive error	40.46 (9.37)	10.18 (8.713)	0.0894	0.9992 (0.0061)	1.220 (0.0625)	0.00663 (0.00048)	1.0154	0.9900 2.731	1.085
Simultaneous estimation, multiplicative errors	1.7340 (0.0252)	1.130 (0.4169)	0.4694	0.6037 (0.0958)	1.238 (0.058)	0.00643 (0.00041)	1.0149	0.9804 0.02186	0.477
Simultaneous estimation, additive errors	58.39 (1.62)	0.4750 (0.1891)	0.6780	0.4471 (0.0134)	1.362 (0.0905)	0.00589 (0.00061)	1.0137	0.9834 3.591	0.618
Simultaneous estimation, less preferred specification of endogenous variables, multiplicative errors	1.7997 (0.00814)	−0.0065 (0.145)	1.0065	0.3391 (0.0312)	1.250 (0.0896)	0.00583 (0.00052)	1.0135	0.9817 0.02114	0.523
Simultaneous estimation, less preferred specification of endogenous variables, additive errors	63.13 (1.22)	−0.0590 (0.142)	1.063	0.3287 (0.0302)	1.255 (0.0907)	0.00648 (0.00056)	1.0150	0.9835 3.585	0.534

Estimates of the Parameters of the Cobb-Douglas Production Function, with Constant Returns to Scale

	A or $\log A$ (1)	α (2)	β (3)	$\alpha + \beta$ (4)	λ (5)	10^{λ} (6)	$\bar{R}^2$ S_u (7)	$\frac{\delta^2}{S^2}$ (8)
Straight regression, multiplicative error	1.9803 (0.0232)	1.031 (0.055)	−0.031 (0.055)	1.00 (. . . .)	0.00805 (0.00025)	1.0187	0.9712 0.01623	0.841
Straight regression, additive error	97.6 (5.90)	1.102 (0.062)	−0.102 (0.062)	1.00 (. . . .)	0.00811 (0.00025)	1.0189	0.9658 3.725	0.762
Simultaneous estimation, multiplicative errors	1.8037 (0.00145)	0.658 (0.0028)	0.342 (0.0028)	1.00 (. . . .)	0.00705 ([illegible])	1.0164	0.9372 0.02489	0.335
Simultaneous estimation additive errors	63.8 (0.73)	0.660 (0.0028)	0.340 (0.0028)	1.00 (. . . .)	0.00745 (0.00035)	1.0173	[illegible] 5.539	0.302

Estimates of the Parameters of the Cobb-Douglas Production Function, with Unconstrained Returns to Scale

	A or $\log A$ (1)	α (2)	β (3)	$\alpha + \beta$ (4)	λ (5)	10^{λ} (6)	$\bar{R}^2$ S_u (7)	$\frac{\delta^2}{S^2}$ (8)
Straight regression, multiplicative error	1.9564 (0.0202)	1.167 (0.0505)	0.035 (0.054)	1.202 (0.048)	0.00696 (0.00033)	1.0162	0.9925 0.01353	1.33
Straight regression additive error	83.31 (5.66)	1.145 (0.0696)	0.062 (0.073)	1.207 (0.064)	0.00690 (0.00043)	1.0160	0.9899 2.807	1.07
Simultaneous estimation, multiplicative errors	1.7947 (0.0014)	0.960 (0.061)	0.496 (0.032)	1.456 (0.093)	0.00484 (0.00054)	1.0112	0.9795 0.02234	0.66
Simultaneous estimation additive errors	61.87 (0.73)	0.964 (0.055)	0.501 (0.034)	1.465 (0.098)	0.00526 (0.00063)	1.0122	0.9811 3.839	0.64

APPENDIX III

NON-LINEAR PROGRAMMING SAMPLE OUTPUT,

PROBLEM CARD AND OUTPUT

Exhibit IX

FORTRAN STATEMENTS

FOR TAYLOR SERIES LINEAR APPROXIMATION

```
      SUBROUTINE FUN(F,D,P ,X)
      REAL D(4), P(4), X(3)
       F=P(1)*((P(2)*(X(1)**(-P(3)))+(1-P(2))*(X(2)**(-P(3))))**(-P(4)))
      G=((P(2)*(X(1)**(-P(3))))+((1-P(2))*(X(2)**(-P(3)))))
      D(1)=(G**(-P(4)))*P(1)
      D(2)=(P(4)*P(1))*(G**(-P(4)-1))*((X(1)**(-P(3)))-(X(2)**(-P(3))))*
     1P(2))
      D(3)=(-P(4)*P(1))*(G**(-P(4)-1))*(P(2)*(X(1)**(-P(3)))*ALOG10(X(1)
     1)+(1-P(3))*(X(2)**(-P(3)))*ALOG10(X(2)))*P(3)
      D(4)=P(1)*(G**(-P(4)))*ALOG10(G)*P(4)
      RETURN
      END
```

Exhibit X

SAMPLE PROBLEM STATEMENT

```
BMDX85 - NON LINEAR LEAST SQUARES - REVISED NOVEMBER 11,1969

HEALTH SCIENCES COMPUTING FACILITY, UCLA

PROBLM CODE                         PRODUT

NUMBER OF VARIABLES                      3

INDEX OF THE DEPENDENT VARIABLE          3

INDEX OF THE WEIGHTING VARIABLE          0

NUMBER OF CASES                         20

NUMBER OF PARAMETERS                     4

TOLERANCE                           0.000010

EPSILON                             0.010000

MAXIMUM NUMBER OF ITERATIONS            20

NUMBER OF VARIABLE FORMAT CARDS          1

ALTERNATE INPUT TAPE NUMBER              5

REWIND OPTION                          NO

VARIABLE FORMAT                     (F7.0,3X,F6.0,3X,F6.0)
```

Exhibit XI

SAMPLE OUTPUT PART A

```
MINIMA                    4.0000E-03  0.0         0.0         1.0000E 00

MAXIMA                    9.0000E-03  1.0000E 00  1.0000E 00  4.0000E 00

ITERATION     ERROR       PARAMETERS
              MEAN
              SQUARE
   0  0     1.0192E 09    4.0000E-03  3.0466E-01  4.8383E-01  3.0245E 00
   1  0     3.4609E 07    9.0000E-03  3.0466E-01  4.9356E-01  3.0177E 00
   2  2     1.8532E 07    9.0000E-03  2.2850E-01  4.9702E-01  3.0177E 00
   3  9     1.7906E 07    9.0000E-03  2.2850E-01  4.9800E-01  3.0171E 00
   4  6     1.7801E 07    9.0000E-03  2.2492E-01  4.9789E-01  3.0171E 00

THE PROCESS IS NOT CONVERGING

ASYMPTOTIC STANDARD DEVIATIONS OF THE PARAMETERS

            0.0           2.7933E-01  1.1079E-02  0.0

ASYMPTOTIC CORRELATION MATRIX OF THE PARAMETERS

               1             2             3             4
   1          0.0           0.0           0.0           0.0
   2          0.0           1.00000       0.90861       0.0
   3          0.0           0.90861       1.00000       0.0
   4          0.0           0.0           0.0           0.0
```

Exhibit XII

SAMPLE OUTPUT PART B

CASE	F	Y-F	STANDARD DEVIATION OF ESTIMATE	VARIABLES		
1	7355.27734	1232.72266	495.93018	57343.00000	6217.00000	8588.00000
2	9490.49219	350.50781	739.58301	63631.00000	7414.00000	9841.00000
3	12253.90625	-904.90625	1063.45239	71465.00000	8837.00000	11349.00000
4	14090.56250	-930.56250	1154.34521	80442.00000	9673.00000	13160.00000
5	15304.30078	-425.30078	1143.73193	88475.00000	10179.00000	14879.00000
6	16625.81641	406.18359	950.82129	103843.00000	10648.00000	17032.00000
7	20324.21094	-367.21094	1177.46118	117797.00000	12180.00000	19957.00000
8	27131.25781	-5106.25781	1514.65894	144086.00000	14750.00000	22025.00000
9	31334.92188	-7033.92188	1566.97290	164350.00000	16181.00000	24301.00000
10	33879.16016	-6147.16016	1512.79443	179302.00000	16990.00000	27732.00000
11	32587.54297	-1607.54297	887.08496	200656.00000	16359.00000	30980.00000
12	34312.69922	492.30078	766.45728	220264.00000	16849.00000	34805.00000
13	38021.39063	2229.60938	830.33521	238022.00000	18027.00000	40251.00000
14	45633.10938	-840.10938	1090.98560	260043.00000	20410.00000	44793.00000
15	49629.45703	1151.54297	1065.63525	287449.00000	21506.00000	50781.00000
16	55962.11719	156.88281	1179.23022	325409.00000	23215.00000	56119.00000
17	66699.00000	-1040.00000	1438.29370	369794.00000	26070.00000	65659.00000
18	79815.93750	-5958.93750	1725.56860	411225.00000	29410.00000	73857.00000
19	80833.75000	4014.25000	2076.75903	451036.00000	29470.00000	84848.00000
20	83239.50000	10252.50000	2557.02246	487195.00000	29924.00000	93492.00000

APPENDIX IV

PREDICTIONS WITH USE OF BMDX85

Exhibit XIII

PREDICTIONS WITH OPTION A

(20 years)

CASE	F	Y-F	STANDARD DEVIATION OF ESTIMATE	VARIABLES		
1	7333.39063	1254.60938	497.88306	57343.00000	6217.00000	8588.00000
2	9462.51172	378.43828	742.20605	63631.00000	7414.00000	9841.00000
3	12217.98828	-868.98828	1066.91089	71465.00000	8837.00000	11349.00000
4	14048.87500	-688.87500	1158.04688	80442.00000	9673.00000	13160.00000
5	15258.51953	-379.51953	1147.38696	88475.00000	10179.00000	14879.00000
6	16574.94922	457.05078	954.01222	103843.00000	10648.00000	17032.00000
7	20261.79688	-304.79688	1181.15259	117797.00000	12180.00000	19957.00000
8	27047.29688	-5022.29688	1518.97266	144086.00000	14750.00000	22025.00000
9	31236.96875	-6935.96875	1571.30762	164350.00000	16181.00000	24301.00000
10	33772.28516	-6040.28516	1516.92773	179302.00000	16990.00000	27732.00000
11	32482.14453	-1502.14453	889.92603	200656.00000	16359.00000	30980.00000
12	34200.60156	604.39844	768.93774	220264.00000	16849.00000	34805.00000
13	37896.42578	2354.57422	832.90112	238022.00000	18027.00000	40251.00000
14	45463.64453	-690.64453	1094.10327	260043.00000	20410.00000	44793.00000
15	49465.35156	1315.64844	1068.67651	287449.00000	21506.00000	50781.00000
16	55775.32031	343.67969	1182.53149	325409.00000	23215.00000	56119.00000
17	66475.37500	-816.37500	1442.39380	369794.00000	26070.00000	65659.00000
18	79547.68750	-5690.68750	1730.54541	411225.00000	29410.00000	73857.00000
19	80559.12500	4288.87500	2082.73193	451036.00000	29470.00000	84848.00000
20	82952.93750	10539.06250	2563.88989	487195.00000	29924.00000	93492.00000

Exhibit XIV

PREDICTIONS WITH OPTION B

(17 years)

CASE	F	Y-F	STANDARD DEVIATION OF ESTIMATE	VARIABLES		
1	7146.72656	1441.27344	395.82251	57343.00000	6217.00000	8588.00000
2	9215.33203	625.66797	601.76074	63631.00000	7414.00000	9641.00000
3	11890.91797	-541.91797	675.99463	71465.00000	8837.00000	11349.00000
4	13669.13672	-509.13672	941.31152	80442.00000	9673.00000	13160.00000
5	14844.34766	34.65234	918.55542	88475.00000	10179.00000	14879.00000
6	16124.89453	907.10547	728.32397	103843.00000	10648.00000	17032.00000
7	19702.56250	254.43750	901.25492	117797.00000	12180.00000	19957.00000
8	26284.17969	-4259.17969	1146.66162	144086.00000	14750.00000	22025.00000
9	30347.63281	-6046.63281	1160.78442	164350.00000	16181.00000	24301.00000
10	32806.85547	-5074.85547	1099.60889	179302.00000	16990.00000	27732.00000
11	31563.34375	-583.34375	681.57813	200656.00000	16359.00000	30980.00000
12	33232.33984	1572.66016	713.06738	220264.00000	16849.00000	34805.00000
13	36815.69531	3435.30469	808.97339	236022.00000	18027.00000	40251.00000
14	44165.94922	627.05078	952.10693	260043.00000	20410.00000	44793.00000
15	48026.52344	2754.47656	1111.13452	287449.00000	21506.00000	50781.00000
16	54142.00391	1976.99609	1429.50366	325409.00000	23215.00000	56119.00000
17	64504.57813	1154.42188	1799.39111	369794.00000	26070.00000	65659.00000

Exhibit XV

PREDICTIONS WITH OPTION C

(16 years)

CASE	F	Y-F	STANDARD DEVIATION OF ESTIMATE	VARIABLES		
1	7065.51953	1502.48047	438.42529	57343.00000	6217.00000	8568.00000
2	9159.49219	681.50781	664.48364	63631.00000	7414.00000	9841.00000
3	11846.18750	-497.18750	965.26050	71465.00000	8837.00000	11349.00000
4	13618.08203	-458.08203	1031.18848	80442.00000	9673.00000	13160.00000
5	14785.94141	93.05859	999.79761	88475.00000	10179.00000	14879.00000
6	16025.55078	1006.44922	781.53638	103843.00000	10648.00000	17032.00000
7	19601.13281	355.86719	960.88721	117797.00000	12180.00000	19957.00000
8	26176.16016	-4151.16016	1208.31616	144096.00000	14750.00000	22025.00000
9	30219.19531	-5918.19531	1212.92407	164359.00000	16181.00000	24301.00000
10	32655.77344	-4923.77344	1147.01343	179302.00000	16990.00000	27732.00000
11	31327.81250	-347.81250	796.28442	200656.00000	16359.00000	30980.00000
12	32955.45313	1849.54688	902.23755	220264.00000	16649.00000	34805.00000
13	36518.77734	3732.22266	1037.40527	238022.00000	18027.00000	40251.00000
14	43870.82813	922.17188	1192.45752	260043.00000	20410.00000	44793.00000
15	47684.19141	3096.80859	1455.51294	287449.00000	21506.00000	50781.00000
16	53740.89844	2378.10156	1901.84961	325409.00000	23215.00000	56119.00000

Exhibit XVI

PREDICTIONS WITH OPTION D

(15 years)

CASE	F	Y-F	STANDARD DEVIATION OF ESTIMATE	VARIABLES		
1	7182.81641	1405.18359	420.38306	57343.00000	6217.00000	8588.00000
2	9287.16406	553.83594	635.78418	63631.00000	7414.00000	9841.00000
3	12013.82422	-664.82422	922.40552	71465.00000	8837.00000	11349.00000
4	13812.64453	-652.64453	987.16235	80442.00000	9673.00000	13160.00000
5	14993.42578	-114.42578	959.45996	88475.00000	10179.00000	14879.00000
6	16257.79688	774.20313	755.36719	103843.00000	10648.00000	17032.00000
7	19888.76563	68.23438	929.32886	117797.00000	12180.00000	19957.00000
8	26567.26563	-4542.26563	1171.30176	144086.00000	14750.00000	22025.00000
9	30675.15234	-6374.15234	1179.11060	164350.00000	16181.00000	24301.00000
10	31803.41016	-823.41016	748.82910	200656.00000	16359.00000	30980.00000
11	33457.83203	1347.16797	826.29272	220264.00000	16849.00000	34805.00000
12	37079.11719	3171.88281	946.43164	238022.00000	18027.00000	40251.00000
13	44550.94922	242.05078	1097.29370	260043.00000	20410.00000	44793.00000
14	48427.34766	2353.65234	1321.64087	287449.00000	21506.00000	50781.00000
15	54585.17188	1533.82813	1718.55005	325409.00000	23215.00000	56119.00000

Exhibit XVII

PREDICTIONS WITH OPTION E

(tolerance reduced - 15 years)

CASE	F	Y-F	STANDARD DEVIATION OF ESTIMATE	VARIABLES		
1	7180.49609	1407.50391	424.57690	57343.00000	6217.00000	8588.00000
2	9291.61719	549.38281	640.08789	63631.00000	7414.00000	9841.00000
3	12028.43359	-679.43359	926.65356	71465.00000	8837.00000	11349.00000
4	13829.91016	-669.91016	991.27515	80442.00000	9673.00000	13160.00000
5	15010.02734	-131.02734	963.50098	86475.00000	10179.00000	14879.00000
6	16266.72266	765.27734	759.60034	103843.00000	10648.00000	17032.00000
7	19906.58594	50.41406	932.87378	117797.00000	12180.00000	19957.00000
8	26600.89063	-4575.89063	1173.14307	144086.00000	14750.00000	22025.00000
9	30713.51563	-6412.51563	1180.11597	164350.00000	16181.00000	24301.00000
10	31812.52344	-832.52344	751.38721	200656.00000	16359.00000	30980.00000
11	33459.08984	1345.91016	827.49731	220264.00000	16849.00000	34805.00000
12	37084.22656	3166.77344	948.20728	238022.00000	18027.00000	40251.00000
13	44576.97656	216.02344	1101.76392	260043.00000	20410.00000	44793.00000
14	48450.00391	2330.99609	1327.03638	287449.00000	21506.00000	50781.00000
15	54607.42188	1511.57813	1725.64355	325409.00000	23215.00000	56119.00000